8.1 notes calculus

Chapter 8
The Integral as Net Change

1. How do we calculate areas that appear to be irregular? (First paragraph) Finite sums (Riemann Sums)
LRAM RRAM MRAM

2. What ideas make it easier to calculate irregular quantities? (Second paragraph) integrals
technology

3. How long have the following methods been around to help compute irregular quantities?
a. finite sums (modeling step) Thousands of years
b. integrals hundreds of years
c. technology tens of years

4. a. If you integrate an acceleration function, what do you get?
b. What are the units on an acceleration function?
c. What are the units on a velocity function?
a) velocity b) $\frac{\text{distance}}{\text{time}^2}$ c) $\frac{\text{distance}}{\text{time}}$

5. a. If you integrate a velocity function, what do you get?
b. What are the units on a position function?
a) distance $\boxed{\text{position}}$ b) distance

6. What type of solution would you get if you integrated a velocity function? $\int v(t)\,dt \Rightarrow$ general solution for position
What if you integrated it with an initial condition?
particular solution (because you found "C")

7. If you integrate a function that represents a rate of change over time, what do you get? Explain.
(i.e. if you integrate ft/sec you get ft, if you integrate \$/mile you get \$)
$\int$ f(t) dt rate of change $\Rightarrow$ get whatever the unit is what is being measured per time

8. Do Examples 1, 2, and Exploration 1. There are different ways to find the final position
What are they? Describe them.

Graphically ; Analytically

$$\int t^2 - \frac{8}{(t+1)^2}\,dt$$

$$\int_0^1 t^2 - 8(t+1)^{-2}\,dt$$

$$\frac{t^3}{3} + \frac{-8(t+1)^{-1}}{-1}$$

$$\frac{t^3}{3} + \frac{8}{t+1}\Big|_0^1$$

$$\left(\frac{1}{3}+4\right) - \left(\frac{0}{3}+\frac{8}{1}\right)$$

$$\frac{13}{3} - \frac{24}{3} = \frac{-11}{3}$$

$-\frac{11}{3}$ moved from original position

$s(0)=9$ $\quad 9 + \frac{-11}{3} = \frac{27-11}{3}$ $\frac{16}{3}$ position

$$\int_0^5 v(t)\,dt$$

$$\frac{t^3}{3} + \frac{8}{t+1}\Big|_0^5$$

$$\left(\frac{125}{3} + \frac{8}{6}\right) - (0+8)$$

$$43 - 8 = 35$$

$s(0)=9$
$9+35 = \boxed{44}$

$\boxed{\text{new position} = \text{initial position} + \text{displacement}}$
definite integral

Exploration 1

$$\int t^2 - \frac{8}{(t+1)^2}\,dt$$

initial condition
$s(0)=9$

$$s(t) = \frac{t^3}{3} + \frac{8}{t+1} + C$$

$$9 = \frac{0^3}{3} + \frac{8}{0+1} + C$$

$$9 = 8 + C$$

$$1 = C$$

$$s(t) = \frac{t^3}{3} + \frac{8}{t+1} + 1$$

$$s(1) = \frac{1^3}{3} + \frac{8}{1+1} + 1$$

$$s(1) = \frac{16}{3}$$

$$s(5) = \frac{5^3}{3} + \frac{8}{5+1} + 1$$

$$s(5) = 44$$

② General solution, find found "C" } finding a particular solution
plugged in the time of interest.

9. What is the difference between displacement and position?
How do you get position from displacement? What do you need?
displacement is how much the position has changed
position - where it is
new position = initial (condition position) + displacement
What do you need = initial condition

10. In your own words, what is a general strategy for modeling and calculating net change?
Integrate + C ; find "C" plug in point of interest
or
integrate to find change + initial condition

11. Read Example 4 down to question a) and b) without looking further on the page.
   a. What type of function are you dealing with? $a(t)$
   b. What is a) asking for? velocity
   c. How do you get the answer to a) according to your general method? integrate acceleration $\int_0^8 a(t)\,dt$ + init cond
   d. What is b) asking for? Describe how you would solve b), then look in the book.

Total distance
integrate $|v(t)|$

12. If you integrate a rate of consumption over time, what should you get?
Read Example 5.

$\int C(t)\,dt$ consumption how much is consumed

1970 to 1980 function is
$C(t) = 2.2 + 1.1^t$
from beginning 1972 to end 1973 2 yrs

$\int_{1972-1970}^{1974-1970} \Rightarrow \int_2^4 C(t)\,dt$
fnint

13. If you integrate a function in gallons per minute over a time in minutes, what unit would the answer have? Read Example 6.

$\int R(t)\,dt \Rightarrow$ GALLONS

$\int_0^{60} R(t)\,dt$
Trapezoidal rule $h = \frac{b-a}{n}$ $\frac{60-0}{12}$
$\frac{1}{2}h[y_0 + 2y_1 + 2y_2 + \ldots + 2y_{n-1} + y_n]$
$\frac{1}{2} \cdot \frac{60}{12}[\ldots\ldots\ldots]$

14. Given work problems you will use the formula W = Fd. What do F and d represent? W=work F=force d=distance
There is also Hooke's Law: F = kx. Describe it. What are the units on k? Newton-meters $\frac{\text{Force units}}{\text{distance}}$ Newtons
What are the units on x? distance So, what are the units on F?
When you integrate F over a length (m) what unit should the answer have? Newton-meters
Force · distance
$\frac{N}{\cancel{m}} \cdot m =$

15. The unit you should have written down to the last question is the unit on a quantity in physics.
What quantity is it? Joules

16. Read Example 7. Can you see how the units work out in the same manner as all the other problems?

$F(x) = kx$

$10 = k \cdot 2$

$5 = k \qquad F = 5x$

$W = F \cdot d$

$\int F(x)\,dx$

$\int_0^4 5x\,dx$

$\frac{5x^2}{2}\Big|_0^4$

$\frac{5(4)^2}{2} - \frac{5(0)^2}{2}$

$\boxed{40\text{ N-m}}$

17. Why did they title this section "The Integral as Net Change"? Explain.

The integral helps you figure out how much the quantity changed