6.1 notes calculus

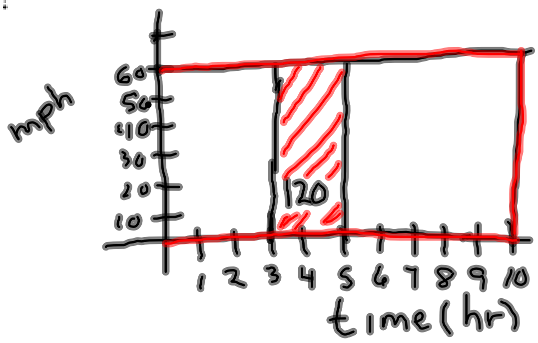
Estimating with Finite Sums

We have studied rates of change, speed, velocity, acceleration, position, displacement values of a function, etc. We have used derivatives and differentials to study these. This can be lumped into one category called differential calculus. We will now turn our attention to accumulation. For instance, if water is entering a tub according to the function $f(t) = 2t^2-4t+1$ ft/min how much water was dumped in between the 3rd and 4th minute. If a particle is moving along the x axis with $v(t) = 2t^3-3$, what is the total distance traveled in the first minute?

Exploration: Find the area of your Hand – inside approximation, outside approximation.

Distance traveled: Suppose we are going along at 60 mph for 10 hours. What is the total distance traveled from hour 3 to hour 5? D= rt

Suppose we wanted to figure out the distance by looking at the velocity graph?



$$\frac{60 \text{ miles}}{1\ \cancel{\text{hr}}} \cdot 2\cancel{\text{hr}} = 120 \text{ miles}$$

The area under the curve between 3 and 5 is exactly 120 miles. What is totally amazing and fascinating is that we can actually use the same method no matter what the curve (function) looks like! The problem is how do we find the area?

Rectangular approximation method (RAM) -- finding the sum of the area of the rectangles to approximate the area of the whole.

MRAM: approximating the area under the curve by finding the sum of the areas of each rectangle that has a base of the length Δt and the height at the midpoint of the subinterval.

LRAM: evaluating the area under the curve using the left-hand endpoint height instead of the midpoint height

RRAM: evaluating the area under the curve using the right-hand endpoint height instead of the midpoint height.

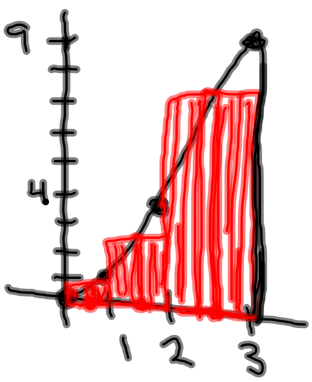
Area under a curve

Example 1

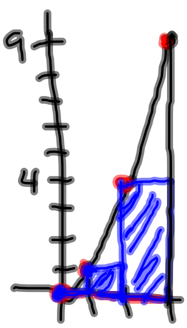
Start with an easy function and the same idea we used for our hand.

A particle starts at x = 0 and moves along the x-axis with velocity v(t) = t^2 for time $t \geq 0$. Where is the particle at t = 3? Draw the graph of v(t) = t^2 for the time interval [0,3] into subintervals of length Δt. Let's use three subintervals. $\frac{3-0}{3}=1$

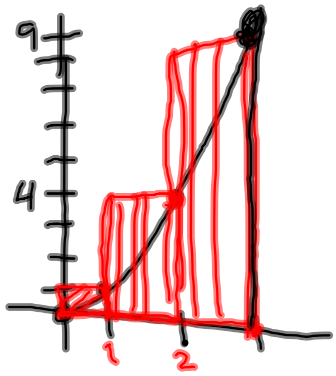
Find MRAM Find LRAM Find RRAM



x	x^2
0	0
1	1
2	4
3	9



LRAM



RRAM

Area of Rectangle

MRAM:

[0,1] [1,2] [2,3]

$1(y(.5)) + 1(y(1.5)) + 1(y(2.5))$

$1\left(\frac{1}{4}\right) + 1\left(\frac{9}{4}\right) + 1\left(\frac{25}{4}\right)$

$\frac{35}{4} = 8\frac{3}{4}$

LRAM:

[0,1] [1,2] [2,3]

$1(y(0)) + 1(y(1)) + 1(y(2))$

$1(0) + 1(1) + 1(4)$

$0 + 1 + 4$

$\boxed{5}$

RRAM:

[0,1] [1,2] [2,3]

$1(y(1)) + 1(y(2)) + 1y(3)$

$1(1) + 1(4) + 1(9)$

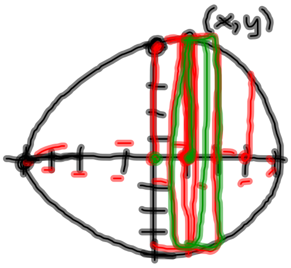
$\boxed{14}$

Example 2: Estimating the area under the graph of a nonnegative function

Read it

Program not on AP EXAM.

Example 3: Estimating the volume of a sphere



$x^2+y^2=r^2$

$x^2+y^2=4^2$

$y^2=16-x^2$

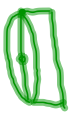
$y=\sqrt{16-x^2}$

$A=\pi r^2$ circle

cylinder $\pi r^2 h$

$V=\pi r^2 h$

see picture in book



y is radius

h is Δx

$\frac{b-a}{n}$ ← subintervals

$\frac{4 - -4}{8}$ $\quad \frac{8}{8}=1$

$V=\pi r^2 h$

LRAM $h=1$ $\quad y=r$ $\quad y=\sqrt{16-x^2}$ $\quad y^2=r^2$

[-4,-3] [-3,-2] [-2,-1] [-1,0] [0,1] [1,2] [2,3] [3,4]

$V=\pi(h)[r^2]$

$$\pi(1)\left[(16-(-4)^2)+(16-(-3)^2)+(16-(-2)^2)+(16-(-1)^2)+(16-(0)^2)+(16-(1)^2)+(16-(2)^2)+(16-(3)^2)\right]$$

LRAM 84π

263.894

True

$V=\frac{4}{3}\pi r^3$ $\quad \frac{4}{3}\pi(4)^3$ $\quad \frac{256\pi}{3}$

≈ 268.083

#17 on homework: Plot and find the area under the curve.

LRAM bh

sec min meter/sec

$60(5)[1+1.2+1.7+2.0+1.8+1.6+1.4+1.2+1.0+1.8+1.5+1.2]$

$60(5)[17.4]$

$60(87) = 5220$ meters

RRAM

$(60)(5)[1.2+1.7+2.0+1.8+1.6+1.4+1.2+1.0+1.8+1.5+1.2+0]$

$300(16.4)$

$\boxed{4920 \text{ meters}}$