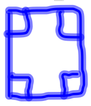
5.4 notes calculus

Modeling and Optimization

When we **optimize** something we maximize or minimize. When we optimize fuel economy we want the maximum number of miles/gallon. When we optimize production each person works we are looking for the maximum efficiency to minimize the cost.

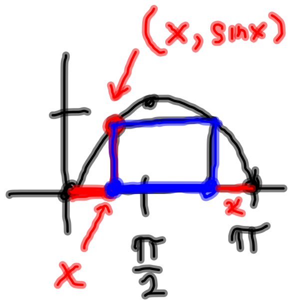
If I gave you a sheet of paper that is 8 ½ x 11 inches; what is the maximum volume of an open top box that can be made with this piece of paper?



How do you find volume of a rectangular prism? V= LWH

Your answer must have justification. You may work in groups. You must have correct dimensions of box, correct volume, ~~must construct the box~~, and **justify your answer by using calculus**. You must show all work. (20 points extra credit)

Example 2: inscribing rectangles



sinx

Domain $[0, \pi]$

maximize area of rectangle

$A = LW$

$A = bh$

height = sinx

base = $\pi - 2x$

$A = (\pi - 2x)(\sin(x))$

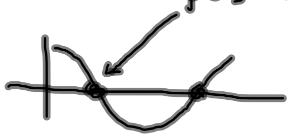
product

$A' = (\pi - 2x)\cos(x) + (\sin(x))(-2)$

$y_1 = (\pi - 2x)(\cos(x)) + -2\sin(x)$

Find 0   window
mode - radians

pos to neg



$x \approx .710463$

height $\sin(.710463) \approx .652$ units

base $\pi - 2(.710463) \approx 1.721$ units

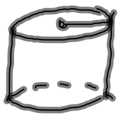
max Area $\approx$ bh $\boxed{1.122 \text{ units}^2}$

Strategies for solving Max and Min problems

1. Understand the problem. Read and identify information needed
2. Develop a mathematical model of the problem. Draw pictures, label parts that are important. Find a variable. Write a function whose extreme value gives the information sought.
3. Graph the function. Find domain. Determine what values of the variable make sense.
4. Identify the critical points and endpoints. Find where the derivative is zero or fails to exist.
5. Solve the model. Support solution with another method if needed.
6. Interpret the solution. Translate your result into the problem setting and decide whether the result makes sense.

Example 4 Designing a Can

minimize material

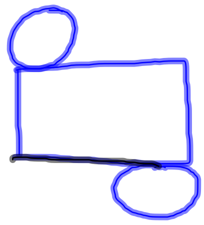


$V = 1$ Liter   1 Liter $= 1000\,cm^3$

$V = \pi r^2 h$

$1000 = \pi r^2 h$

$\frac{1000}{\pi r^2} = h$



Surface Area $= 2\pi r^2 + 2\pi r h$

$A = 2\pi r^2 + 2\cancel{\pi r}\left(\frac{1000}{\cancel{\pi r^2}_{r}}\right)$

$A = 2\pi r^2 + \frac{2000}{r}$ $\frac{d}{dr}\frac{2000}{r}$ $2000r^{-1}$

$-2000r^{-2}$

$A' = 4\pi r - \frac{2000}{r^2}$

$0 = 4\pi r - \frac{2000}{r^2}$

$\frac{2000}{r^2} = 4\pi r$

$\frac{2000}{4\pi} = \frac{4\pi r^3}{4\pi}$

$\sqrt[3]{\frac{500}{\pi}} = r$ $\boxed{r = \sqrt[3]{\frac{500}{\pi}}\ cm}$

2nd deriv test $r \approx$

$A' = 4\pi r - 2000r^{-2}$

$A'' = 4\pi + 4000r^{-3}$

$A'' = 4\pi + \frac{4000}{r^3}$ concave up

A'' is always positive   min

$r = \sqrt[3]{\frac{500}{\pi}}$ cm

$r \approx 5.419$ cm

$h = \frac{1000}{\pi r^2}$ $\frac{1000}{(\pi)(5.419)^2} \approx 10.8395$

Theorem 6: Maximizing Profit

Maximum profit (if any) occurs at a production level at which marginal revenue equals marginal cost.

Example 5

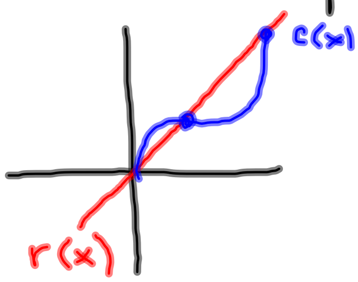
$R(x)$ revenue $\qquad C(x)$ cost

marginal revenue $\qquad$ maginal cost

$R'(x) \qquad C'(x)$

$r(x) = 9x \qquad c(x) = x^3 - 6x^2 + 15x$

$r'(x) = c'(x)$



$9 = 3x^2 - 12x + 15$

$0 = 3x^2 - 12x + 6$

$\approx .586$ max loss

≈ 3.414 cost below revenue max profit

Theorem 7: Minimizing Average Cost
The production level (if any) at which average cost is smallest is a level at which the average cost equals the marginal cost. $\to C'(x)$

Example 6

average cost $\frac{C(x)}{x}$

$C(x) = x^3 - 6x^2 + 15x$ $C'(x) = 3x^2 - 12x + 15$

av cost $\frac{x^3 - 6x^2 + 15x}{x}$

av cost $x^2 - 6x + 15$

av cost $= C'$

$x^2 - 6x + 15 = 3x^2 - 12x + 15$

$0 = 2x^2 - 6x$

x is in thousands $0 = 2x(x-3)$

$x = 0$ $x = 3$

when I make 3,000

To make sense of economics mathematics let's read - Modeling Discrete Phenomena with Differentiable Functions