5.3 notes calculus

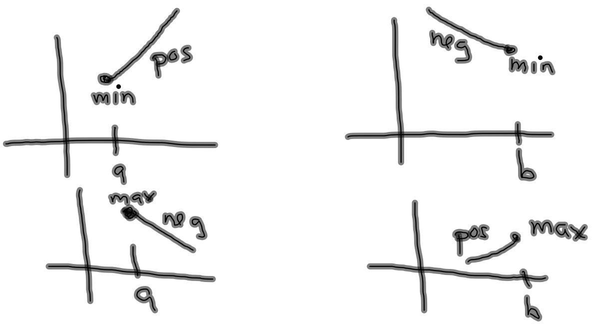
Connecting f′ and f″ with the graph of f

We already know that when:
f′ (c) = 0 c is a possible max or min
f′ (c) > 0 at c, *f* is increasing
f′ (c) < 0 at c, *f* is decreasing

Look at Figure 5.18
p209 analyze graph

at a is abs min – lowest point on graph
at c_1 $f'(c_1)=0$ not an extrema
no sign change
at c_2 $f'(c_2)=0$ sign change pos to neg max
at c_3 $f'(c_3)=0$ Sign change neg to pos min
at c_4 $f'(c_4)=$ dne cusp abs max
critical pt.
at c_5 $f'(c_5)=0$ no sign change not extrema
at b endpoint local min

Check out: First Derivative Test for Local Extrema
If f ′ changes sign from positive to negative at c then f has a local maximum value at c.
If f ′ changes sign from negative to positive at c then f has a local minimum value at c.
If f ′ does not change sign at c, then f has no local extreme value at c.
At a left endpoint a: If f ′ < 0 for x>a, then f has a local maximum value at a. If f ′ > 0 for x > a, then f has a local minimum value at a.
At a right endpoint b: If f ′ < 0 for x<b, then f has a local minimum value at a. If f ′ > 0 for x < b, then f has a local maximum value at a.
p209-210

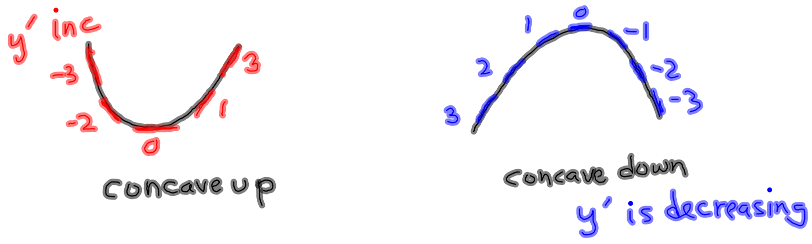


Look at figure 5.21

There is a nice definition of concavity.

Concave up when y′ is increasing

Concave down when y′ is decreasing

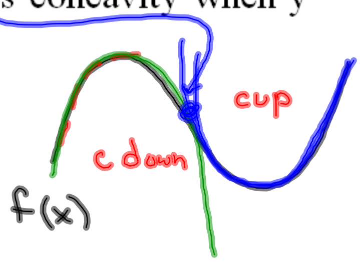


We are really talking about how y′ is changing.

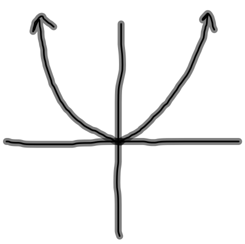
Concave up when $y'' > 0$ when y'' is positive

Concave down when $y'' < 0$ when y'' is neg

Changes concavity when $y'' = 0$



Now look at page 212 figure 5.22 and 5.23



$y = x^2$

$y' = 2x$ $y' = 0$ at $x = 0$

$y'' = 2$ always positive

	$(-\infty, 0)$	$(0, \infty)$
y'	$-$	$+$
y	dec	inc

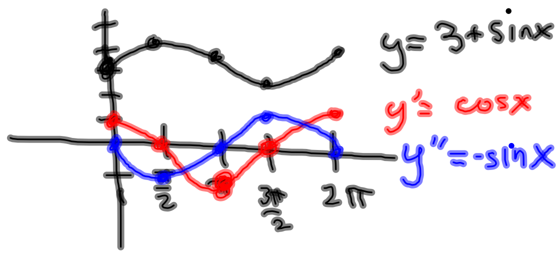
	$(-\infty, \infty)$
y''	pos
y'	inc

figure 5.23

$y = 3 + \sin x$

$y' = \cos x$

$y'' = -\sin x$



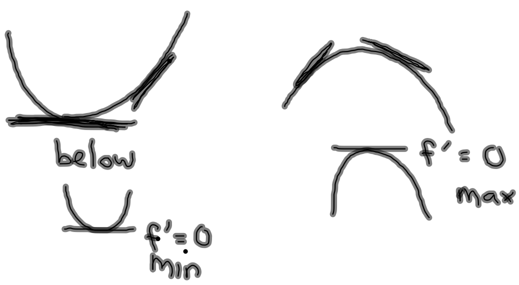
Points of Inflection

A **point of inflection** is a point where the graph of a function has a tangent line and where the concavity changes.

Critical Points

f′ (c) Does not exist (in the function this is at a corner, cusp, jump, not if the point is a vertical tangent)
f′ (c) = 0 possible max or min
Remember that max or min can occur at an endpoint of the interval.
f″ (c) = 0 point of inflection. Changes from concave up to down or down to up; but it must have a tangent line.

It is good to note that when a function is increasing (concave up) the tangent line is below the curve. When it is concave down, the tangent line is above the curve.



Example:

$y = x^3 - 12x - 5$ $y' = 3x^2 - 12$ $y'' = 6x$

Test the critical points:
f′ (2) = 0 and f″(2)>0 then f has a local minimum at x = 2
f′ (-2) = 0 and f″(-2)<0 then f has a local maximum at x = -2
f′ (0) < 0 and f″(0)=0 then point of inflection

$y = x^3 - 12x - 5$ D: $(-\infty, \infty)$
$y' = 3x^2 - 12$
$y' = 3(x^2 - 4)$
$y' = 3(x-2)(x+2)$
$0 = 3(x-2)(x+2)$
$y'' = 6x$ $6x = 0$

max −2, min 2
$(-\infty,-2)$ | $(-2,2)$ | $(2,\infty)$
y′: + − +
y: inc dec inc

0
$(-\infty, 0)$ | $(0,\infty)$
y″: − +
y: c down cup

max ___ at x=−2
$y' = 0$ at $x = -2$ and $y'' < 0$ at $x = -2$

min ___ at x=2
$y'(2) = 0$ and $y''(2) > 0$ so min at $x = 2$

Example:

$y = x^3 - 12x - 5$ $y' = 3x^2 - 12$ $y'' = 6x$

Find intervals where function is increasing and decreasing

Intervals	x<-2 $(-\infty,-2)$	X=-2	-2< x<0 $(-2,0)$	0<x<2 $(0,2)$	X=2	x>2 $(2,\infty)$
f′	Positive	0	Negative	negative	0	positive
f″	Negative	negative	Negative	positive	positive	positive
Function	Increasing cd	positive	decreasing cd	decreasing cup	negative	increasing cup

Second Derivative Test for Local Extrema

If $f'(c) = 0$ and $f''(c) < 0$ then f has a local maximum at $x = c$

If $f'(c) = 0$ and $f''(c) > 0$ then f has a local minimum at $x = c$

Example 8

$f'(x) = 4x^3 - 12x^2$

$0 = 4x^2(x-3)$

inc $(3, \infty)$

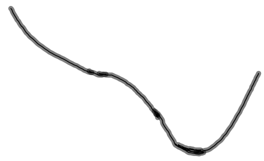
dec $(-\infty, 3)$

$f''(x) = 12x^2 - 24x$

$0 = 12x(x-2)$

	$(-\infty,0)$	0	$(0,3)$	3 (min)	$(3,\infty)$
f'	$-$		$-$		$+$
f	dec		dec		inc

	$(-\infty,0)$	0	$(0,2)$	2	$(2,\infty)$
f''	$+$		$-$		$+$
f	cup		cd		cup



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$f'(x) = 4x^3 - 12x^2$

$f(x) = \frac{4x^{3+1}}{(3+1)} - \frac{12x^{2+1}}{(2+1)}$

$f(x) = x^4 - 4x^3$

$f(x) = x^4 - 4x^3 + 7$

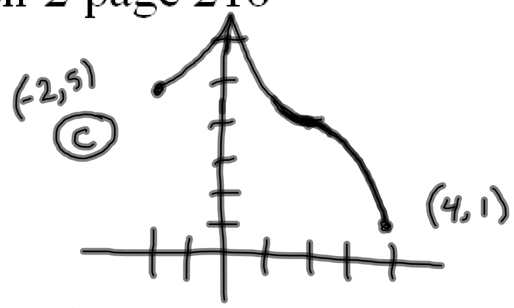
$f(x) = x^4 - 4x^3 - 3$

vertical shift

Learn about functions from Derivatives.

Read the Table on pg 217

Exploration 2 page 218



c

a) abs min $(4,1)$

abs max at $x=0$

b) pt of inflection at $x=2$

Examples:
Use analytic methods to find the intervals on which the function is increasing, decreasing, concave up and concave down. Also find local extreme values, and inflection points.

3. $y = 2x^4 - 4x^2 + 1$ D: $(-\infty, \infty)$ no end pts

$y' = 8x^3 - 8x$
$y' = 8x(x^2 - 1)$
$y' = 8x(x+1)(x-1)$
$0 = 8x \quad x+1=0 \quad x-1=0$

min max min
$(-\infty,-1) \mid (-1,0) \mid (0,1) \mid (1,\infty)$
y': $-$ $+$ $-$ $+$
y: dec inc dec inc

max 1 at $x=0$
min -1 at $-1=x$
min -1 at $x=1$

inc $(-1,0)\cup(1,\infty)$
dec $(-\infty,-1)\cup(0,1)$

$y'' = 24x^2 - 8$
$y'' = 8(3x^2-1)$
$0 = 8(3x^2-1)$
$0 = 3x^2 - 1$
$1 = 3x^2$
$\frac{1}{3} = x^2$
$\pm\sqrt{\frac{1}{3}} = x$

$-\sqrt{\frac{1}{3}}$ $\sqrt{\frac{1}{3}}$
$(-\infty,-\sqrt{\frac{1}{3}}) \mid (-\sqrt{\frac{1}{3}},\sqrt{\frac{1}{3}}) \mid (\sqrt{\frac{1}{3}},\infty)$
y'': $+$ $-$ $+$
y: cup cd cup

y concave up $(-\infty,-\sqrt{\frac{1}{3}})\cup(\sqrt{\frac{1}{3}},\infty)$
concave down $(-\sqrt{\frac{1}{3}},\sqrt{\frac{1}{3}})$

INFLECTION POINT (x_1, y_1)

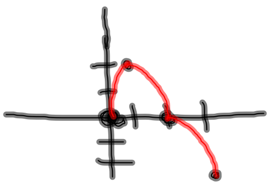
$\left(-\sqrt{\frac{1}{3}},\ 2\left(-\sqrt{\frac{1}{3}}\right)^4 - 4\left(-\sqrt{\frac{1}{3}}\right)^2 + 1\right)$

$\left(\sqrt{\frac{1}{3}},\ 2\left(\sqrt{\frac{1}{3}}\right)^4 - 4\left(\sqrt{\frac{1}{3}}\right)^2 + 1\right)$

51. f is continuous on [0,3] and satisfies the following. Find the absolute extrema of f and where they occur; find any points of inflection, sketch a possible graph of f.

x	0	1	2	3
f	0	2	0	-2
f′	3	0	does not exist	-3
f″	0	-1	does not exist	0

x	0<x<1	1<x<2	2<x<3
f	+	+	-
f′	+	-	-
f″	-	-	-



a) max (1,2)
min (3,-2)
b) none