5.1 Notes calculus

Extreme Values of Functions

Calculus is about rates of change and accumulation. We have studied rates of change (derivatives). That branch of mathematics is called differential calculus. Since we are now good at differentiating functions, let's learn to apply derivatives in useful ways.

In the past, one of the main uses of the derivative was to find information about a graph. This is generally called finding critical values of a graph. But we will do much more than that with derivatives.

If we talk about extreme temperatures, what are we talking about? high - max low - min

In calculus we are concerned with when a function reaches an absolute max or absolute min. (absolute = global)

Graphically, what do max and min have in common?

If the function has a local maximum value or local minimum value at an interior point of its domain and if $f'(c)$ exists, then $f'(c) = 0$

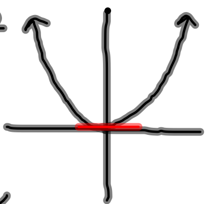
Look at Figure 5.2. p192

$y = x^2$

$y' = 2x$

$0 = 2x$

$\boxed{0 = x}$ $y(0) = 0^2$ y-value



D: $(-\infty, \infty)$

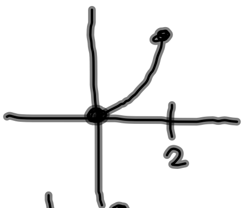
abs max : none

abs min: 0 at $x=0$

$y = x^2$ D: $[0, 2]$

abs max 4 at $x=2$

abs min 0 at $x=0$



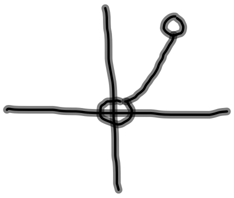
$y = x^2$

D: $(0, 2]$

abs max 4 at $x=2$

abs min: none

$y = x^2$



D: $(0, 2)$

abs max none

abs min none

Theorem 1: The Extreme Value Theorem
If *f* is continuous on a closed interval [a, b], then f has both a maximum value and a minimum value on the interval.

A critical point(s) of a function occurs when:
***f ' (c)* does not exist**
***f ' (c)* = 0 possible point of max or min occurs**
Extreme values occur only at critical points and endpoints.
Never forget that a max or min can occur at an end point.

Definition: A point in the interior of the domain of a function *f* at which *f '* (c) = 0 is called a stationary point.

#17 $f(x) = x^{\frac{2}{5}} \quad -3 \leq x < 1$

$f'(x) = \frac{2}{5}x^{-\frac{3}{5}}$

endpoint $x = -3$
$f(-3) = (-3)^{\frac{2}{5}}$ $\boxed{\sqrt[5]{9}}$

$f'(x) = \frac{2}{5x^{\frac{3}{5}}}$

$f'(x) = \frac{2}{5\sqrt[5]{x^3}}$

$0 = \frac{2}{5\sqrt[5]{x^3}}$ never

when does $f'(x) = 0$
when $f'(x)$ not exist
when denominator = 0

$5\sqrt[5]{x^3} = 0$
when $x^3 = 0$
$x = 0$

$f'(x)$ dne at $x = 0$
$f(0) = 0^{\frac{2}{5}}$

Abs max or min
$f(0) = 0$ — $f'(x)$ dne
$f(-3) = \sqrt[5]{9}$ — endpoint — max
max is $\sqrt[5]{9}$ at $x = -3$
min is 0 at $x = 0$

Example: extreme values: $f(x)=\dfrac{1}{\sqrt{4-x^2}}$

Find them and say max or min. How is the slope changing on either side?

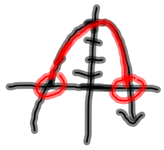
Find the domain $(-2,2)$

denominator

$4-x^2>0$

$-x^2+4=0$

$(2-x)(2+x)$



No end points. if function has a max or min then it must occur at critical point

$f(x)=1(4-x^2)^{-\frac{1}{2}}$

$f'(x)=-\frac{1}{2}\cdot 1(4-x^2)^{-\frac{1}{2}-1}\cdot -2x$

$-\frac{1}{2}\cdot 1\cdot -2x$

$f'(x)=x(4-x^2)^{-\frac{3}{2}}$

$f'(x)=\dfrac{x}{\sqrt{(4-x^2)^3}}$

$f'(x)=\dfrac{x}{(4-x^2)\sqrt{4-x^2}}$

where does $f'(x)=0$ at $x=0$

$0=\dfrac{x}{(4-x^2)\sqrt{4-x^2}}$

where does the denominator $=0$

$f'(x)$ dne at $x=2$ and $x=-2$

not in domain

so only critical point is at $x=0$

$f(0)=\dfrac{1}{\sqrt{4-0^2}}$ $\boxed{\frac{1}{2}\ \text{min}}$

sign chart

	$(-2,0)$	0	$(0,2)$
$f'(x)$	$-$		$+$

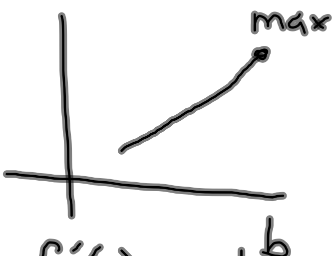
Look at page $\overset{209}{210}$ Theorem 4: The first derivative test for local extrema

[a, b]

left end point

$f'(x)<0$ to the right

right end point



$f'(x)>0$ to the left

Examples:

$y = x^2 + 4x$ $\quad D: (-\infty, \infty)$ $\quad y' = 2x + 4$

$0 = 2x + 4$

$x = -2$

min -4 at $x = -2$

$(-\infty, -2) \; \overset{-2}{|} \; (-2, \infty)$

$f'(x)$ $\quad - \quad +$

min

$y = x^3$ $\quad y' = 3x^2$

$0 = 3x^2$

$(-\infty, 0) \; \overset{0}{|} \; (0, \infty)$

$+ \quad +$ no sign change, no extrema

$y = x^{\frac{1}{3}}$ $\quad y' = \frac{1}{3}x^{-\frac{2}{3}}$ $\quad \frac{1}{3\sqrt[3]{x^2}}$

$0 = y'$ never $\quad$ dne

at $x = 0$

$(-\infty, 0) \; \overset{0}{|} \; (0, \infty)$

$f'(x)$ $\quad + \quad +$

no extrema

$f(x) = x^3 - 12x - 5$

$f(x) = (x^2 - 3)e^x$

$f(x) = x^3 - 12x - 5$ $\quad D: (-\infty, \infty)$

$f'(x) = 3x^2 - 12$

$3(x^2 - 4)$

$f'(x) = 3(x-2)(x+2)$

$0 = 3(x-2)(x+2)$

$(-\infty, -2) \; \overset{-2}{|} \; (-2, 2) \; \overset{2}{|} \; (2, \infty)$

max $\quad$ min

$f'(x)$ $\quad + \quad - \quad +$

min -21 at $x = 2$

max 11 at $x = -2$

$f(2) = 2^3 - 12(2) - 5$ $\quad f(2) = -21$

$f(-2) = (-2)^3 - 12(-2) - 5$ $\quad f(-2) = 11$

$f(x) = (x^2 - 3)\, e^x \qquad D: (-\infty, \infty)$

$f'(x) = (x^2 - 3)\, e^x + e^x (2x)$

$e^x x^2 - 3e^x + 2xe^x$

$f'(x) = e^x (x^2 + 2x - 3)$

$f'(x) = 0$

will e^x ever $= 0$? No

$f'(x) = e^x (x + 3)(x - 1)$

$0 = e^x (x + 3)(x - 1)$

critical points $x = -3$ $x = 1$

	$(-\infty, -3)$	max -3	$(-3, 1)$	min 1	$(1, \infty)$
$f'(x)$	+	$\cap$	−	$\cup$	+

max of $\underline{.299}$ at $x = -3$

min of ___ at $x = 1$

$f(-3) = ((-3)^2 - 3)\, e^{-3} \qquad .299$

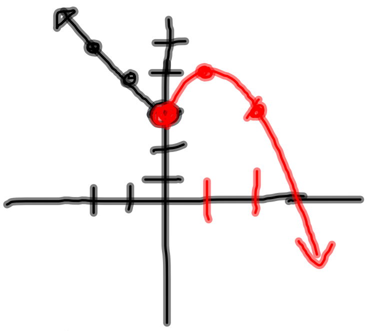
$f(1) = (1^2 - 3)\, e^1 \qquad -5.437$

This will be tied together by section 5.3

#40 page 198

$$y = \begin{cases} 3 - x & x < 0 \\ 3 + 2x - x^2 & x \geq 0 \end{cases}$$

x	$3+2x-x^2$
0	3 closed circle
1	$3+2-1$
2	$3+4-4$



$y' = -1 \quad x < 0$

$y' = 2 - 2x \quad x \geq 0$

$f'(x)$ dne at $x = 0$

$f'(x) = 0$ at

$y' = 2 - 2x$

$0 = 2 - 2x$

$2x = 2$

$x = 1$

min max

0 1

	$(-\infty, 0)$	$(0, 1)$	$(1, \infty)$
$f'(x)$	$-$	$+$	$-$

max of 4 at $x = 1$

min of 3 at $x = 0$

#5

abs max $x = b$

abs min $x = c_2$

Closed interval

$f'(x) = 0$ at c_2

$f'(x) > 0$ to left of the right endpoint

Do #26 without a calculator:

#26 $y = \dfrac{1}{\sqrt[3]{1-x^2}}$ D: $(-\infty, \infty)$ except $x = \pm 1$

$y = 1(1-x^2)^{-\frac{1}{3}}$

$y' = -\frac{1}{3}(1-x^2)^{-\frac{4}{3}} \cdot -2x$

$y' = \frac{2}{3}x(1-x^2)^{-\frac{4}{3}}$

$y' = \dfrac{2x}{3\sqrt[3]{(1-x^2)^4}}$

when $y' = 0$ at $x = 0$

when y' dne at $x = \pm 1$ but $x \neq \pm 1$

$$
\begin{array}{c|cccc}
 & (-\infty,-1) & (-1,0) & (0,1) & (1,\infty) \\
 & \text{und } -1 & 0 & \text{und } 1 & \\
y' & - & - & + & +
\end{array}
$$

min at $x = 0$

$y(0) = \dfrac{1}{\sqrt[3]{1-0^2}} = 1$