3.5 notes calc

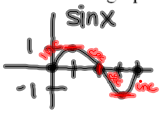
Derivatives of Trigonometric Functions

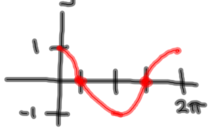
Trig functions are important because so many things that occur in nature are periodic. So we need to know the derivatives of the trigonometric functions.

Graph: $y = \sin(x)$

Estimate the slope of the curve at various points to graph y′ from y.

What does this graph look like? y′ cos x





$\frac{d}{dx}\sin(x) = \boxed{\cos x}$

What does this graph look like?

Prove this to be true.

$$\frac{d}{dx}\sin x = \lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h} = \frac{\sin(x)\cos(h)+\cos(x)\sin(h)-\sin(x)}{h}$$

$$= \frac{\sin(x)\cos(h)-\sin(x)+\cos(x)\sin(h)}{h} = \frac{\sin(x)[\cos(h)-1]+\cos(x)\sin(h)}{h}$$

split into product

$$= \lim_{h\to 0}\frac{\sin x(\cos(h)-1)}{h} + \lim_{h\to 0}\frac{\cos(x)\sin(h)}{h}$$ split into product

$$\lim_{h\to 0}\sin(x)\cdot\lim_{h\to 0}\frac{\cos(h)-1}{h} + \lim_{h\to 0}\cos(x)\cdot\lim_{h\to 0}\frac{\sin(h)}{h}$$

$$= \lim_{h\to 0}\sin(x)\cdot 0 + \lim_{h\to 0}\cos(x)\cdot 1 = \cos(x)$$

$$\therefore \frac{d}{dx}\sin x = \cos(x)$$

mult by 0 = 0

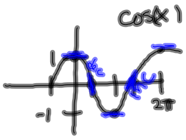
$\lim_{h\to 0}\frac{\sin(h)}{h} = 1$

$\lim_{x\to 0}\frac{\sin x}{x} = 1$

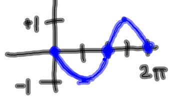
Graph: $f = \cos(x)$

Estimate slope.

What does this look like?



$\frac{d}{dx}\cos(x) = -\sin x$



$$\boxed{\frac{d}{dx}\cos(x) = -\sin(x)}$$

Use these derivatives and rules to do more complicated derivatives.

Example: $y = x^2\sin(x)$ $y' =$

Product

$x^2\left(\frac{d}{dx}\sin(x)\right) + \sin x\cdot\left(\frac{d}{dx}x^2\right)$

$x^2\cos(x) + \sin(x)\cdot 2x$

$\boxed{x^2\cos(x) + 2x\sin(x)}$

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$$y = \frac{x}{1+\cos(x)} \qquad y' =$$

Quotient Rule

$$\frac{[1+\cos(x)](1) - x\left[\frac{d}{dx}(1+\cos x)\right]}{(1+\cos(x))^2}$$

(0 − sin x)

$$\frac{1+\cos(x) - x(-\sin x)}{(1+\cos(x))^2}$$

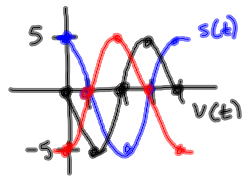
$$\boxed{\frac{1+\cos(x)+x\sin(x)}{(1+\cos(x))^2}}$$

Simple Harmonic Motion Application (like a pendulum)
Example 2 p143

$S(t) = 5\cos t$

$V(t) = -5\sin t$

$a(t) = -5\cos t$



Other trigonometric derivatives Memorize these!

$\frac{d}{dx}\tan(x) = \sec^2(x)$ $\qquad$ $\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$

$\frac{d}{dx}\cot(x) = -\csc^2 x$ $\qquad$ $\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$

PSST

positive sec sec tan

$\frac{d}{dx}\sec x = \sec x \tan x$

$\frac{d}{dx}\tan x = \sec^2 x$
$(\sec x)^2$

NC$_{sc}$ Csc Cot

$\frac{d}{dx}\csc x = -\csc x \cot x$

$\frac{d}{dx}\cot x = -\csc^2 x$
$-(\csc x)^2$

Jerk

A *sudden change in acceleration* is called a "*jerk*". When a ride in a car or a bus is jerky, it is not that the accelerations involved are necessarily large, but that the changes are abrupt. Jerk is what spills your soft drink. The derivative responsible for jerk is the third derivative of position. Jerk is the derivative of acceleration.

$S(t)$ position

$S'(t) = V(t)$ velocity

$S''(t) = V'(t) \rightarrow a(t)$ accel.

$S'''(t) = V''(t) = a'(t) \Rightarrow$ jerk

If a body's position at time t is s(t), the body's jerk at time t is:

$$j(t)=\frac{da}{dt}=\frac{d^3s}{dt^3}$$

Remember:

$s = position$ $\quad \frac{ds}{dt} = velocity$ $\quad \frac{d^2s}{dt^2} = acceleration$

$\frac{d^3s}{dt^3} = jerk \ or \ change \ in \ acceleration$

Jerks can describe why we get motion sick.

Find jerk of going down the road when v=50 mph.
$a =$ 0 jerk = 0

Find jerk when we are standing around.
$a = 32$ ft/sec^2 j= 0

In Example 2
s= 5cos(t) v= -5sin(t) a= -5cos(t) da/dt = j = 5sin(t)
The greatest magnitude (changes) do not occur at the extremes of the displacement, but at the rest position, where the acceleration changes direction and sign.
Example: Scrambler at LaGoon!

We have and can find 1st, 2nd, 3rd, 4th ....etc. derivatives.
Also, tangent and normal lines as well.

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$y = \sin x + 3$ at $x = \pi$

$(\pi, 3)$
Find y_1
$y = \sin\pi + 3$
$y = 0 + 3$

Tangent line equation
$y - y_1 = m(x - x_1)$
$y - 3 = m(x - \pi)$

need slope at $x = \pi$
$y' = \cos x$
$y'(\pi) = \cos(\pi)$
-1

$y - 3 = -1(x - \pi)$
$y = -x + \pi + 3$

Normal line
$y - 3 = m(x - \pi)$ opp reciprocal of -1 → 1
$y - 3 = 1(x - \pi)$
$y = x - \pi + 3$