3.4 notes calc

Velocity and Other Rates of Change

In general, ***using the difference quotient*** *and* ***taking the limit as $h \to 0$*** will give us the ***derivative***, a formula for the ***tangent to the curve***, or the ***instantaneous rate of change***. Knowing what the rate of change is at any given point in time can be very useful in medicine, in factory production, in economics, and so forth.

at a point

Recall, when we were interested at a specific point $x = a$ we found $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$. Even if the function doesn't relate to time, we still say ***instantaneous rate of change***. In fact, whenever we say rate of change it is understood ***from now on*** that we mean instantaneous rate of change.

Example: Find the rate of change of the area of a circle with respect to the radius.
How do we write this?

The change in y with respect to x was written $\frac{\Delta y}{\Delta x}$ or $\frac{dy}{dx}$.
Here it is area (A) with respect to radius r, so we need to write it $\frac{dA}{dr}$. We need a function that relates the two. What is the formula for finding the area of a circle? $A = \pi r^2$ What is its derivative? $\frac{dA}{dr} = 2\pi r$

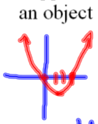
Evaluate the rate of change for r = 5 and r = 10

$2\pi(5)$ $2\pi(10)$ 20π

10π

What units would be appropriate for $\frac{dA}{dr}$? $\frac{\text{units}^2}{\text{units}}$

Motion along a Line
Linear motion, position graphs s or s(t), velocity graph v(t) or s′, acceleration graph a(t) or s″
Suppose we have a position function that tells us the position of an object with respect to time: s=f(t) Example: $s(t) = t^2-3t$



$y_2 - y_1$

The ***displacement*** of the object over the time interval from t to t+Δt, or net change in position is $\Delta s = f(t+\Delta t)-f(t)$ and the ***average velocity*** of the object over that time interval is

$$v_{av} = \frac{displacement}{travel\ time} = \frac{\Delta s}{\Delta t} = \frac{f(t+\Delta t)-f(t)}{\Delta t}$$

Does this formula look familiar?

Slope formula!

Instantaneous velocity is the derivative of the position function s=f(t) with respect to time. At time t the velocity is

$$v(t) = \frac{ds}{dt} = \lim_{\Delta t\to 0}\frac{f(t+\Delta t)-f(t)}{\Delta t}$$

In our example position function of $s(t) = t^2-3t$ find the derivative or in other words the velocity. $2t - 3$
What is the velocity at t=3? $2(3)-3 = \boxed{3}$

Example 2

Read

Pg 129

Back to $s(t) = t^2-3t$ Find velocity at t=1. $s'(1) = 2(1) -3$ $s''(1) = -1$

Sometimes velocity is positive and sometimes velocity is negative. Velocity not only tells us how fast, it also tells us which direction.

☆ *Speed* is the absolute value of velocity. $\boxed{\text{Speed} = |v(t)| = |ds/dt|}$

negative velocity means the position moving to the left.
positive velocity means position moving to the right.
zero velocity means the position is stopped.

Example 3 Pg129

If we take the 2^{nd} derivative of the position, we get the acceleration or rate of change of the velocity.
The rate at which a body's velocity changes is called the body's acceleration. The acceleration measures how quickly the body picks up or loses speed.

Definition: Acceleration
Acceleration is the ***derivative of velocity with respect to time***. If a body's velocity at time *t* is $v(t) = ds/dt$, then the body's acceleration at time *t* is $a(t) = dv/dt = d^2s/dt^2$.

Remember Free-fall constants (on Earth)
English units: (s in feet) $s = 16t^2$ $g = a = 32$ ft/sec^2

Metric units: (s in meters) $s = 4.9t^2$ $g = a = 9.8$ m/sec^2

#13 page 136

H.W

part a)
$s = 24t - .8t^2$
$v(t) = 24 - 1.6t$ m/sec
$a(t) = -1.6$ m/sec^2

part b)
what is the slope at a maximum? zero
$v(t) = 0$
$0 = 24 - 1.6t$ and solve
$24/1.6 = 15$ so t = 15 sec

part c)
How high did rock go. Plug is 15 sec into s(t)
$s(15) = 24(15) - .8(15)^2$
180 meters

part d)
when did rock reach 1/2 of maximum.
use calculator
$24t - .8t^2 = 90$ then find intersection points

$t \approx 4.393$ sec and $t \approx 25.607$

part e)
How long is the rock in motion?
$0 = 24t - .8t^2$
30 seconds

Example 4 & 5

Pg 130

Remember:
$v(t) < 0$ means the position function is moving to the left
$v(t) > 0$ means the position function is moving to the right
$v(t) = 0$ means the position function is stop

Exploration #2 and #3

Read and follow the instructions for each exploration

The first derivative allows us to see how sensitive a function is to change. If d/dx is constant, it's not sensitive. If it's not, it can have varying degrees of sensitivity.

When a small change in x produces a large change in the value of a function we say that the function $f(x)$ is relatively sensitive to changes in x. The derivative $f'(x)$ is a measure of this sensitivity.

In other words the steeper the curve the more sensitive to change the function is.

Example 6

Derivatives in Economics

As we have seen $s' = velocity = \frac{ds}{dt}$ and $s'' = v' = acceleration = \frac{d^2s}{dt^2}$

The derivative of a cost function gives us the marginal cost, the rate at which the cost changes as production amount changes.

#27 page 138. $C'(x) =$ marginal cost

$C(x) = 2000 + 100x - .1x^2$

average cost of x=100

$$\frac{C(100)}{100}$$

$$\frac{2000 + 100(100) - .1(100)^2}{100} = \boxed{\$110}$$

C'(x)=100-0.2x
C'(100)=100-0.2(100)

c) $$\frac{C(101) - C(100)}{101-100}$$

$$\frac{11,079.90 - 11,000}{1} = \boxed{\$79.90}$$