3.3 notes calculus

Rules for Differentiation

Constant Rule $\frac{d}{dx}(c)=0$

Example: $f(x)=19 \quad f'=0$ f(x) = 7 f'(x) = 0

Power Rule $\frac{d}{dx}x^n = n\cdot x^{n-1}$

(power as long as x ≠ 0 and/or h ≠ 0)

Example:

$f(x)=x^2 \quad f'(x)=2x$

$2x^{2-1}$

$f(x)=x^4 \quad f'(x)=4x^3$

$4x^{4-1}$

$f(x)=x^7 \quad f'(x)=$

$7x^{7-1}$

$\boxed{7x^6}$

$f(x)=\sqrt{x} \qquad f'(x)=\frac{1}{2\sqrt{x}}$

$x^{\frac{1}{2}} = \frac{1}{2}x^{\frac{1}{2}-1}$

$\frac{1}{2}x^{-\frac{1}{2}}$

$\frac{1}{2}\cdot\frac{1}{x^{1/2}}$

$\frac{1}{2x^{\frac{1}{2}}} = \frac{1}{2\sqrt{x}}$

Constant Multiple Rule $\frac{d}{dx}k\cdot u = k\cdot\frac{du}{dx}$

Example:

$f(x)=5x^2 \quad f'(x)=10x$

$5\cdot\frac{d}{dx}(x^2)$

$5\cdot 2x$

$10x$

$f(x)=-2x^4 \quad f'(x)=-8x^3$

$f(x)=2x^5 \quad f'(x)=$ $\boxed{10x^4}$

Sum/Difference Rule $\frac{d}{dx}(u\pm v)=\frac{du}{dx}\pm\frac{dv}{dx}$

Example:

$f(x)=x^3+5x^2-\frac{2}{5}x-4 \qquad f'(x)=3x^2+10x-\frac{2}{5}$

$y=3t^2-4t+2 \quad \frac{dy}{dt}=6t-4$

Product Rule $\frac{d}{dx}(u\cdot v)=u\frac{dv}{dx}+v\frac{du}{dx}$

(1st times deriv of 2nd + 2nd times deriv of 1st)

$p=(x^2+3x)(x-1) \qquad \frac{dp}{dx}=$

$(x^2+3x)(1) + (x-1)(2x+3)$

$x^2+3x + 2x^2+3x-2x-3$

$\boxed{3x^2+4x-3}$

Quotient Rule $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\cdot\frac{du}{dx}-u\cdot\frac{dv}{dx}}{v^2}$

(bottom times deriv top – top times deriv bottom all over bottom squared)

$f(x)=\frac{2x}{x-1} \qquad f'(x)=\frac{(x-1)\frac{d}{dx}(2x)-(2x)\frac{d}{dx}(x-1)}{}$

$\frac{(x-1)2-2x(1)}{(x-1)^2}$

$\frac{2x-2-2x}{(x-1)^2}$

$\frac{-2}{(x-1)^2}$

$f(x)=\frac{1}{x-1} \qquad f'(x)=$

$\frac{(x-1)(0)-1(1)}{(x-1)^2}$

$\frac{-1}{(x-1)^2}$

OR $(x-1)^{-1}$

$-1(x-1)^{-1-1}$

$-1(x-1)^{-2}$

$\frac{-1}{(x-1)^2}$

What if you are only given the value of the functions and their derivatives at certain points?

Let $y = u \cdot v$ $u(2) = 3$ $u'(2) = -4$ $v(2)=1$ $v'(2) = 2$
Find $y'(2)$

By product rule:

$y'(2) = u(2)\,v'(2) + v(2)\,u'(2)$ $y'(2) = 3(2) + 1(-4)$ $y'(2) = 2$

If $y = \frac{u}{v}$ *Find* $\frac{dy}{dx}(2)$

$\frac{v(2)u'(2) - u(2)v'(2)}{v(2)^2}$

$\frac{1(-4) - 3(2)}{1^2} = \frac{-4-6}{1} \Rightarrow \boxed{-10}$

If we can differentiate a function once, what's to stop us from differentiating again.
f' is 1st derivative, f'' (f double prime) is 2nd derivative
f''' (f triple prime)

$\frac{dy}{dx}$ $\frac{d^2y}{dx^2}$ $\frac{d^3y}{dx^3}$ "d squared y d x squared" "d cubed y d x cubed"

y' 1st derivative y'' 2nd derivative y''' 3rd derivative
y^n nth derivative

Example:
$f(x) = 2x^4 - x^3 + 6x$
$f'(x) = 8x^3 - 3x^2 + 6$
$f''(x) = 24x^2 - 6x$
$f'''(x) = 48x - 6$
$f''''(x) = 48$
$f'''''(x) = 0$

Recall position (s), velocity(v), and acceleration (a)
S(t) $s = 16t^2$ v(t) $s' = 32t$ a(t) $s'' = 32$ ft/sec^2