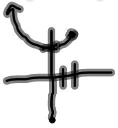
Calculus Practice Exam Chapter 1 -1 1

Directions: Show all steps leading to your answers, including any intermediate results obtained using a graphing utility.



1. Let $f(x)=\begin{cases} x^2+5, & x\le 2 \\ \dfrac{4x-3}{x+3}, & x>2 \end{cases}$

Find the limit of $f(x)$ as

a) $x\to-\infty$;

$\lim_{x\to-\infty}$

$\boxed{\infty}$

b) $x\to 2^-$;

$\lim_{x\to 2^-}$

2^2+5

$\boxed{9}$

c) $x\to 2^+$;

$\lim_{x\to 2^+}$

$\dfrac{4(2)-3}{2+3}$

$\dfrac{8-3}{5}=\dfrac{5}{5}$

$\boxed{1}$

d) $x\to\infty$

$\lim_{x\to\infty} f(x)$

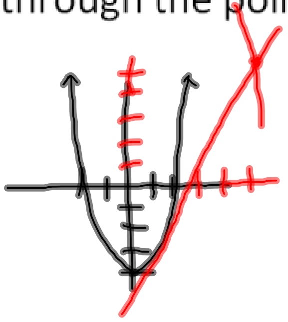
$\dfrac{4x-3}{x+3}$

H.A.

$\boxed{4}$

or L'Hopital

2. Find the equations of all lines tangent to $y = x^2 - 4$ that pass through the point (5, 5).



x_1, y_1 $\quad$ x_2, y_2

(a, a^2-4)

tangent line $y - y_1 = m(x - x_1)$

$y - 5 = m(x - 5)$

Find slope

2 points $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad \frac{a^2-4-5}{a-5} = \frac{a^2-9}{a-5}$

and

1st derivative of $y = x^2 - 4$

$\frac{dy}{dx} = 2x$

if $x = a$ $\quad 2a = m$

so

$2a = \frac{a^2-9}{a-5}$

$2a(a-5) = a^2 - 9$

$2a^2 - 10a = a^2 - 9$

$a^2 - 10a + 9 = 0$

$(a-1)(a-9) = 0$

$a - 1 = 0 \quad a - 9 = 0$

$a = 1 \quad a = 9$

Slope of tangent line $\quad 2x \quad 2(1) = 2$

$2x \quad 2(9) = 18$

2 tangent lines

$$\boxed{\begin{array}{l} y - 5 = 2(x-5) \\ y - 5 = 18(x-5) \end{array}}$$

3. Find $\frac{dy}{dx}$, where $y = \frac{x^2-5}{\cos(x)}$

quotient rule

$$\boxed{\frac{\cos(x)\cdot(2x)-(x^2-5)(-\sin x)}{\cos^2(x)}}$$

$$2x\cos(x)+(x^2-5)\sin(x)$$

4. Use implicit differentiation to find

$\frac{dy}{dx}$, if $x^2(3y-5)=y^3-x$

y is a function of x

$$x^2(3y-5)=y^3-x$$

distribute

$$(3x^2)y-5x^2=(y)^3-x$$

product rule

chain rule

$$3x^2\cdot\frac{dy}{dx}+y(6x)-10x=3y^2\cdot\frac{dy}{dx}-1$$

$$3x^2\frac{dy}{dx}-3y^2\frac{dy}{dx}=-1-6xy+10x$$

$$\frac{dy}{dx}(3x^2-3y^2)=-1-6xy+10x$$

$$\boxed{\frac{dy}{dx}=\frac{-1-6xy+10x}{3x^2-3y^2}}$$

5. Find $\frac{dy}{dx}$, if $y = 4^{-x}$

A. $(-x)(4^{-x-1})$ B. -4^{-x} C. $(\ln 4)^{-x}$

D. $(\ln 4)(4^{-x})$ [✓ E.] $(-\ln 4)(4^{-x})$

4^{-x} is an exponential function

$$\frac{d}{dx}(a^u) = a^u \cdot \ln a \cdot \frac{du}{dx}$$

$a=4 \quad u=-x$

$a^u = 4^{-x}$

$$\boxed{4^{-x} \cdot \ln 4 \cdot -1}$$

6. For, $y = x - 3e^{-x^2}$ use graphing techniques with analytical support to find the approximate intervals on which the function is:

(a) increasing; $y' > 0$ y' is pos (b) decreasing $y' < 0$ y' is neg

(c) concave up $y'' > 0$ (d) concave down $y'' < 0$

(e) local extreme values, (f) inflection points

min $y' = 0$

neg to pos ∪

max $y' = 0$

pos to neg ∩

$y'' = 0$ (x_1, y_1)

need sign change

$$y' = 1 - 3e^{-x^2} \cdot -2x$$

$$y' = 1 + 6xe^{-x^2}$$

$$y'' = 6x \cdot e^{-x^2} \cdot -2x + e^{-x^2} \cdot 6$$

$$y'' = -12x^2e^{-x^2} + 6e^{-x^2}$$

a) $(-\infty, -1.477) \cup (-.172, \infty)$

b) $(-1.477, -.172)$

c) $(-.707, .707)$

d) $(-\infty, -.707) \cup (.707, \infty)$

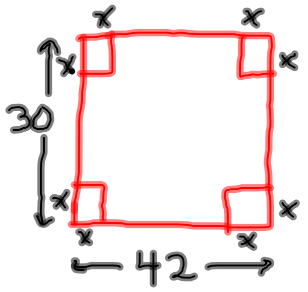
e) min -3.085 at $x = -.172$

max -1.816 at $x = -1.477$

f $(-.707, -2.527)$

$(.707, -1.112)$

7. You are planning to make an open box from a 30- by 42-inch piece of sheet metal by cutting congruent squares from the corners and folding up the sides. You want the box to have the largest possible volume. Using calculus determine the dimensions of the box that would give the largest possible volume of your box. Also find the maximum volume. (Remember the units.)

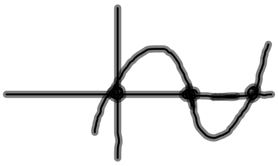


$V = LWH$

$V = x(30-2x)(42-2x)$

Domain of real world problem

$(0, 15)$



$V = (30x - 2x^2)(42-2x)$

$V = 1260x - 60x^2 - 84x^2 + 4x^3$

$V = 4x^3 - 144x^2 + 1260x$

$V'(x) = 12x^2 - 288x + 1260$

$V'(x) = 12(x^2 - 24x + 105)$

quad formula

$$x = \frac{24 \pm \sqrt{(-24)^2 - 4(1)(105)}}{2(1)}$$

$$x = \frac{24 \pm \sqrt{156}}{2}$$

$$\frac{24 \pm 2\sqrt{39}}{2}$$

$12 + \sqrt{39} \approx 18.245$ too big

$12 - \sqrt{39} \approx 5.755$ in

$h \approx 5.755$ in

$L \approx 30.49$ in

$W \approx 18.49$ in

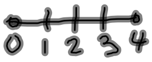
$V = 3244.439 \text{ in}^3$

$L \approx 42 - 2(5.755)$

$W \approx 30 - 2(5.755)$

8. Consider the integral $\int_0^4 (x^2 - 3x + 5)\ dx$

(a) Estimate the value of the integral using 4 right-endpoint rectangles (RRAM).



(b) Estimate the value of the integral using the Trapezoidal Rule with n = 4.

(c) Integrate to find the exact value of the integral.

RRAM

a) [0,①] [1,②] [2,③] [3,④]

rectangle   use the integrand

x^2-3x+5

bh $(1-0)\overset{y(1)}{(3)}+1\overset{y(2)}{(3)}+1\overset{y(3)}{(5)}+1\overset{y(4)}{(9)}$

$3+3+5+9=\boxed{20}$

x	x^2-3x+5
0	5
1	3
2	3
3	5
4	9

ⓑ area of trapezoids

$\frac{1}{2}h(b_1+b_2)$

$\frac{1}{2}(1)[y(0)+2(y(1))+2(y(2))+2(y(3))+1y(4)]$

$\frac{1}{2}[5+2(3)+2(3)+2(5)+9]$

$\frac{1}{2}[5+6+6+10+9]$

$\frac{1}{2}(36) \Rightarrow \boxed{18}$

ⓒ integrate

$\int_0^4 x^2-3x+5\ dx$

$\frac{x^3}{3}-\frac{3x^2}{2}+5x\Big|_0^4$

$\left[\left(\frac{4^3}{3}-\frac{3(4)^2}{2}+5(4)\right)-(0-0+0)\right]$

$\boxed{\frac{52}{3}}$

9. Find $\frac{d}{dx}\int_0^{2x}(5-t)\ dt$

short cut

$(5-2x)\cdot\frac{d}{dx}(2x)$

$(5-2x)(2)$

$\boxed{10-4x}$

long way

$\int_0^{2x}(5-t)\,dt$

$5t-\frac{t^2}{2}\Big|_0^{2x}$

$5(2x)-\frac{(2x)^2}{2}-(0-0)$

$10x-\frac{4x^2}{2}$

$10x-2x^2$

$\frac{d}{dx}(10x-2x^2)$

$\boxed{10-4x}$

10. Use the Fundamental Theorem of Calculus to evaluate $\int \sqrt{5-\sin(x)}\ \ dx$

A. $\frac{-\cos(x)}{2\sqrt{5-\sin(x)}} + C$

[✓ B.] $\int_0^x \sqrt{5-\sin(t)}\ \ dt + C$

C. $\int_x^0 \sqrt{5-\sin(t)}\ \ dt + C$

D. $\int_0^x \frac{-\cos(x)}{2\sqrt{5-\sin(x)}} + C$

E. $\frac{2}{3}(5-\sin(x))^{\frac{3}{2}} + C$

$$\int_0^x \sqrt{5-\sin(t)}\ \ dt \quad + C$$

11. Solve the initial value problem.

$\frac{dy}{dx} = (3x-4)^3, y(1) = -1$ when $x=1$ $y=-1$

$\int dy = \int (3x-4)^3 dx$

$u = 3x-4$

$du = 3\ dx$

$\frac{1}{3} du = dx$

$\int u^3 \cdot \frac{1}{3} du$

$\frac{1}{3} \cdot \frac{u^4}{4}$

$y = \frac{(3x-4)^4}{12} + C$

$-1 = \frac{1}{12}(3 \cdot 1 - 4)^4 + C$

$-1 = \frac{1}{12}(1) + C$

$-1 - \frac{1}{12} = C$

$-\frac{13}{12} = C$

$$\boxed{y = \frac{1}{12}(3x-4)^4 - \frac{13}{12}}$$

12. Solve the differential equation by separation of variables.

$$\frac{dy}{dx} = \frac{x\cos(x^2)}{y^2-2}$$

Let $u = x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$\int (y^2-2)\,dy = \int x\cos(x^2)\,dx$$

$$\int \frac{1}{2}\cos u\,du$$

$$\frac{1}{2}\sin(x^2)$$

$$\boxed{\frac{y^3}{3} - 2y = \frac{1}{2}\sin(x^2) + C}$$

13. Use integration by parts to evaluate $\int x\sin(2x-5)\quad dx$

$$\int x\ \sin(2x-5)\ \ dx$$

deriv		$\int$
x	+	$\sin(2x-5)$
1	−	$-\frac{1}{2}\cos(2x-5)$
0		$-\frac{1}{4}\sin(2x-5)$

$\int \sin(2x-5)$

$-\cos(2x-5)$

$$\boxed{-\frac{1}{2}x\cos(2x-5)+\frac{1}{4}\sin(2x-5)+C}$$

14. Suppose a certain element has a half-life of 4.6 weeks. If a sample contains 300 grams of this element, how much of it will remain after 7 weeks?

0	300
4.6	150

$A = Pe^{rt}$

$A = A_0 e^{kt}$ find k first

$150 = 300 e^{4.6k}$

$\frac{1}{2} = e^{4.6k}$

$\ln(\frac{1}{2}) = 4.6k$ ~~ln e~~

$\frac{\ln(.5)}{4.6} = k$

$A = A_0 e^{kt}$ $t = 7$

$A = 300e^{(\frac{\ln .5}{4.6} \cdot 7)}$

$\boxed{A \approx 104.480 \text{ g}}$

15. Use Euler's method to solve the initial value problem. Do the first 3 iterations by hand.

Start at $x_0 = 0$ with dx = 0.1 . $\frac{dy}{dx}$ $\frac{-1}{4}(x + y^3), y(0) = 1$

$y = y_1 + m\,(x - x_1)$

$x_0 = 0 \quad y_0 = 1$

$x_1 = .1 \quad y_1 = .975$

$x_2 = .2 \quad y_2 = .9493$

$x_3 = .3 \quad y_3 = .9229$

$$y_{n+1} = y_n + f'(x_n, y_n)\,dx$$

$$y_1 = y_0 + (-\tfrac{1}{4}(x_0 + y_0^3)(.1)$$

$$y_1 = 1 + (-\tfrac{1}{4}(0 + 1^3))(.1)$$

$$y_1 = .975$$

$$y_2 = y_1 + (-\tfrac{1}{4}(x_1 + y_1^3))(.1)$$

$$y_2 = .975 + (-\tfrac{1}{4}(.1 + .975^3)(.1)$$

$$y_3 = .9493$$

$$y_3 = y_2 + (-\tfrac{1}{4}(x_2 + y_2^3))(.1)$$

$$y_3 = .9493 + (-\tfrac{1}{4}(.2 + .9493^3))(.1)$$

$$y_3 = .9229$$

16. The function v(t) = 2t - 4 is the velocity in ft./sec of a particle moving along the x-axis for $0 \leq t \leq 8$. Use analytic methods to answer the following questions.

(a) Determine when the particle is moving to the right, to the left, and stopped.

(b) Find the particle's displacement for the given time interval.

(c) Find the total distance traveled by the particle.

a) right $v(t) > 0$

left $v(t) < 0$

stopped $v(t) = 0$

$2t-4>0$ $\quad t>2$ $\boxed{[2,8]}$ sec

$2t>4$

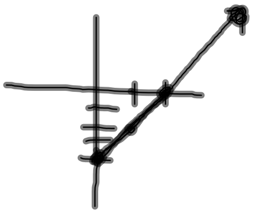
$2t-4<0$ $\quad 2t<4$

$t<2$ $\boxed{[0,2)}$ sec

$2t-4=0$

$2t=4$

$\boxed{t=2}$ sec stopped



b) displacement (net area)

$$\int_0^8 2t-4 \; dt$$

$$\frac{\cancel{2}t^2}{\cancel{2}} - 4t \Big|_0^8$$

$$(8^2 - 4(8)) - (0-0)$$

$$64 - 32$$

$$\boxed{32 \text{ ft}}$$

c) Total distance $\int_0^8 |2t-4| \; dt$

$$-\int_0^2 2t-4 \; dt + \int_2^8 2t-4 \; dt$$

$$-\left(t^2-4t\Big|_0^2\right) + t^2-4t\Big|_2^8$$

$$-[(4-8)-(0-0)] \qquad (8^2-4(8))-(4-8)$$

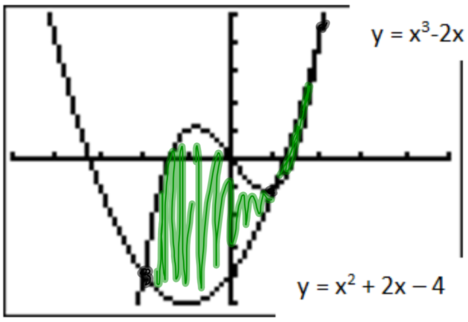
$$4 \qquad 32 - -4$$

$$36$$

$$4+36$$

$$\boxed{40 \text{ ft}}$$

17. Find the area of the shaded region analytically.



on top $(-2,1)$ cubic
$(1,2)$ quad
top - bottom

$$\int_{-2}^{1}(x^3-2x)-(x^2+2x-4)\,dx$$

$$\int_{-2}^{1}x^3-2x-x^2-2x+4\,dx$$

$$\frac{x^4}{4}-\frac{x^3}{3}-\frac{\overset{2}{\cancel{4}}x^2}{\cancel{2}}+4x\Big|_{-2}^{1}$$

$$\left(\frac14-\frac13-2+4\right)-\left(4+\frac83-8-8\right)$$

$$\frac14-\frac13+2-4-\frac83+16$$

$11\frac14$ $\boxed{\frac{45}{4}}$

Find intersections

$$x^3-2x=x^2+2x-4$$

$$(x^3-x^2)(-4x+4)=0$$

$$x^2(x-1)-4(x-1)=0$$

$$(x^2-4)(x-1)=0$$

$$x=-2,\ x=2\quad x=1$$

and

$$\int_1^2(x^2+2x-4)-(x^3-2x)$$

$$x^2+2x-4-x^3+2x$$

$$\int_1^2 x^2+4x-x^3-4$$

$$\downarrow$$

$$\frac{x^3}{3}+2x^2-\frac{x^4}{4}-4x\Big|_1^2$$

$$\left(\frac83+\cancel{8}-4\cancel{-8}\right)-\left(\frac13+2-\frac14-4\right)$$

$$\frac83\cancel{-4}-\frac13-2+\frac14\cancel{+4}$$

$$\frac73+\frac14-2$$

$$\frac{7}{12}$$

Total area $\boxed{\frac{45}{4}+\frac{7}{12}}$

18. A curve is given by $y = f(x)$ for $a \le x \le b$, where $a > 0$ and $f(x) > 0$.

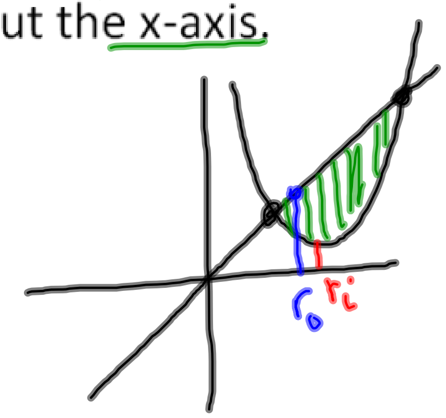
The integral $\int_a^b \sqrt{1+(f'(x))^2}\ \mathrm{dx}$ can be used to find which of the following?

[✓ A.] The length of the curve

B. The volume of the solid generated by revolving the region below the curve about the x-axis

C. The volume of the solid generated by revolving the region below the curve about the y-axis

19. A region is bounded by the line y = x and the parabola $y = x^2 - 6x + 10$. Find the volume of the solid generated by revolving the region about the x-axis.



$y_1 = x$

$y_2 = x^2 - 6x + 10$

$x = x^2 - 6x + 10$

$0 = x^2 - 7x + 10$

$(x-5)(x-2)$

$x = 5 \quad x = 2$

top - bottom

$\pi r_o^2 - \pi r_i^2$

$\pi \int_2^5 x^2 - (x^2 - 6x + 10)^2 \, dx$

use fnint

$\pi \text{ fnint}(y_1^2 - y_2^2, x, 2, 5)$

$\boxed{23.4\pi}$

$\frac{117}{5}\pi$

20. Find the length of the curve describe by $y = \frac{4}{3}x^{\frac{3}{2}}$

for $0 \leq x \leq 6$.

$$\frac{dy}{dx} = \frac{4}{\cancel{3}}\,\frac{\cancel{3}}{2}x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = 2x^{\frac{1}{2}}$$

$$\left(\frac{dy}{dx}\right)^2 = 4x$$

$$L = \int_a^6 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$L = \int_0^6 \sqrt{1+4x}\, dx$$

$$\int_0^6 (1+4x)^{\frac{1}{2}}$$

u-sub

$u = 1+4x$

$du = 4\,dx$

$\frac{1}{4}du$

$$\int_{u(0)}^{u(6)} u^{\frac{1}{2}} \cdot \frac{1}{4}\,du$$

$$\frac{1}{4}\cdot\frac{2}{3}u^{\frac{3}{2}}\Bigg|_{u(0)}^{u(6)}$$

$$\frac{1}{6}u^{\frac{3}{2}}\Bigg|_{1}^{25}$$

$$\frac{1}{6}(25)^{3} - \frac{1}{6}(1)^{\frac{3}{2}}$$

$$\frac{125}{6} - \frac{1}{6} \quad \boxed{\frac{124}{6}} = \frac{62}{3}$$

21. A spring has a natural length of 12 ~~c~~m. A 40-N force stretches the spring to 16 ~~c~~m. How much work is done in stretching the spring from 12 cm. to 24 ~~c~~m.

$F = kx$

$40 = (16-12)k$

$40 = 4k$

$10 = k$

$$\int_{12-12}^{24-12} 10x \, dx$$

$$\int_0^{12} 10x \, dx$$

$$\left.\frac{10x^2}{2}\right|_0^{12}$$

$5(144) = 720$ N-m

22. Use L'Hopital's rule to evaluate each limit.

(a) $\lim\limits_{x\to3}\frac{x^3-2x^2-5x+6}{x^2+2x-15}$ $\frac{3^3-2(3)^2-5(3)+6}{3^2+2(3)-15}$ $\frac{0}{0}$

$\lim\limits_{x\to3}\frac{3x^2-4x-5}{2x+2}$ $\frac{3(3)^2-4(3)-5}{2(3)+2}$ $\frac{27-12-5}{6+2}$ $\boxed{\frac{10}{8}}$

OR $\boxed{\frac{5}{4}}$

(b) $\lim\limits_{x\to\infty}\left(1+\frac{2}{5x}\right)^{3x} = L$

$\lim\limits_{x\to\infty}\ln\left(1+\frac{2}{5x}\right)^{3x} = \ln L$

$\lim\limits_{x\to\infty} 3x\ln\left(1+\frac{2}{5x}\right)$

$\lim\limits_{x\to\infty}\frac{3\ln\left(1+\frac{2}{5x}\right)}{\frac{1}{x}}$

$\frac{d}{dx}\left(1+\frac{2}{5x}\right)$

$\frac{d}{dx}\left(1+\frac{2}{5}\cdot\frac{1}{x}\right)$

$\frac{2}{5}\ \frac{-1}{x^2}$

$$\frac{\frac{3}{1+\frac{2}{5x}}\cdot\frac{-2}{5x^2}}{\frac{-1}{x^2}}$$

$$\frac{-6}{5x^2\left(1+\frac{2}{5x}\right)} \div \frac{-1}{x^2}$$

$$\frac{-6}{5x^2+2x}\cdot\frac{x^2}{-1}$$

$\lim\limits_{x\to\infty}\frac{6x^2}{5x^2+2x}$

$\frac{6}{5}=\ln L$

$\boxed{e^{\frac{6}{5}}}$

23. Order the functions from the slowest-growing to the fastest growing as $x \to \infty$. $e^{\frac{x}{2}}$, $\ln(x+3)$, $5x^2, x^3$

$\ln(x+3),\ 5x^2,\ x^3,\ e^{\frac{x}{2}}$

24. Evaluate the improper integral $\int_2^{\infty} \frac{dx}{x(\ln x)^4}$ or state that it diverges. Show work that justifies your answer.

$$\lim_{b\to\infty} \int_2^b \frac{1}{(\ln x)^4} \cdot \frac{1}{x}\,dx$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$\lim_{b\to\infty} \int_{u(2)}^{u(b)} u^{-4}\ du$$

$$\frac{u^{-3}}{-3}$$

Replaced u with lnx

$$\lim_{b\to\infty} \left.\frac{-1}{3(\ln x)^3}\right|_2^b$$

$$\lim_{b\to\infty} \cancel{\frac{-1}{3(\ln b)^3}} \xrightarrow{} 0 - \frac{-1}{3(\ln 2)^3}$$

$$\frac{1}{3(\ln 2)^3} \approx 1.001$$

25. Express $\frac{(x-2)^2}{(x-1)^2(x^2+3)}$ as a sum of partial fractions.

linear repeated irreducible quadratic

$$\frac{(x-2)^2}{(x-1)^2(x^2+3)} = \boxed{\frac{A}{(x-1)^1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{(x^2+3)}}$$

26. Evaluate $\int \frac{4x^3-69x-83}{(x+1)(x-5)}dx$

x^2-4x-5

improper fraction
long division

$$\begin{array}{r} 4x+16 \\ x^2-4x-5\overline{)4x^3+0x^2-69x-83} \\ {}^{-}4x^3+16x^2+20x \\ \hline 16x^2-49x-83 \\ {}^{-}16x^2+64x+80 \\ \hline 15x-3 \end{array}$$

$\boxed{4x}(x^2-4x-5)$

$\boxed{16}(x^2-4x-5)$

$$\int 4x+16+\frac{15x-3}{(x+1)(x-5)}\,dx$$

partial fraction decomp

$$\frac{15x-3}{(x+1)(x-5)} = \frac{A}{x+1}+\frac{B}{x-5}$$

$$15x-3 = A(x-5)+B(x+1)$$

let $x=5$ $\quad 75-3=6B$

$72=6B \quad B=12$

let $x=-1$ $\quad -18=-6A \quad A=3$

$$\int 4x+16+\frac{3}{x+1}+\frac{12}{x-5}$$

$$\boxed{\frac{4x^2}{2}+16x+3\ln|x+1|+12\ln|x-5|+C}$$

27. Tell whether the series converges or diverges. If it converges, find the sum.

$$5-\frac{15}{4}+\frac{45}{16}-\frac{135}{64}+\ldots+5\left(-\frac{3}{4}\right)^n+\ldots$$

geometric $r: -\frac{3}{4}$ $\left|-\frac{3}{4}\right|<1$ $\boxed{\text{Converges}}$

sum $\frac{a}{1-r}$ $\boxed{\frac{5}{1--\frac{3}{4}}}$ or $\boxed{\frac{5}{\frac{7}{4}}}$ or $\boxed{\frac{20}{7}}$

28. Find the MacLaurin series for xe^{2x}

Include the first three terms and the nth term.

$$e^x = 1+x+\frac{x^2}{2!}+\cdots+\frac{x^n}{n!}+\cdots$$

Replace x with 2x

$$e^{2x} = 1+2x+\frac{(2x)^2}{2!}+\cdots+\frac{(2x)^n}{n!}+\cdots$$

mult each term by x

$$xe^{2x} = x+2x^2+\frac{x(2x)^2}{2!}+\cdots+\frac{x(2x)^n}{n!}+\cdots$$

OR

$$x+2x^2+\frac{4x^3}{2!}+\cdots+\frac{2^n x^{n+1}}{n!}+\cdots$$

OR

$$x+2x^2+2x^3+\cdots+\frac{2^n x^{n+1}}{n!}+\cdots$$

29. Let f(x) = (3x – 2)^8. Use the Remainder Estimation Theorem to estimate the maximum absolute error when f(x) is replaced by 256 – 3072x for $|x| \le 0.04$.

$f(x)=(3x-2)^8$ approx by 256 − 3072x

first order

so $|R_n(x)| \le \frac{f^{n+1}(c)(x-a)^{n+1}}{(n+1)!}$

$$|R_1(x)| \le \frac{f^2(c)\,(x-a)^2}{2!}$$

$$|R_1(x)| \le \frac{504(3x-2)^6(x)^2}{2!}$$

$$\frac{504[3(-.04)-2)^6(-.04)^2}{2!}$$

$$\boxed{36.605}$$

$a=0$ centered around 0

$f(x)=(3x-2)^8$

$f'(x)=8(3x-2)^7(3)$

$24(3x-2)^7$

$f''(x)=168(3x-2)^6\cdot 3$

$504(3x-2)^6$

Let x = .04

OR x = −.04

find max

$504(3(.04)-2)^6$

$504(3(-.04)-2)^6$

30. Determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{(n-3)}{n^3+\ln(2n)}$. Justify your answer by identifying and using the test (or tests) you use.

$\frac{(n-3)}{n^3+\ln(2n)}$ looks like $\frac{1}{n^2}$ $\quad \sum_{n=1}^{\infty} \frac{1}{n^2}$ converges by Pseries

$$\lim_{n\to\infty} \frac{\frac{n-3}{n^3+\ln(2n)}}{\frac{1}{n^2}}$$

$$\lim_{n\to\infty} \frac{(n-3)}{n^3+\ln(2n)} \cdot \frac{n^2}{1}$$

$$\lim_{n\to\infty} \frac{n^2(n-3)}{n^3+\ln(2n)}$$

$$\lim_{n\to\infty} \frac{n^3-3n^2}{n^3+\ln(2n)} \Rightarrow 1$$

By LCT since $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges

$\sum_{n=1}^{\infty} \frac{n-3}{n^3+\ln(2n)}$ converges

DCT $\quad \sum_{n=1}^{\infty} \frac{1}{n^2}$ Converges By p-Series

$$\frac{n-3}{n^3+\ln(2n)} \le \frac{1}{n^2}$$

$$\frac{n-3}{n^3+\ln(2n)} \le \frac{1}{n^2} \cdot \frac{n}{n}$$

$$\frac{n-3}{n^3+\ln(2n)} \le \frac{n}{n^3} \quad \text{True}$$

By DCT $\sum_{n=1}^{\infty} \frac{n-3}{n^3+\ln(2n)}$ converges

31. Which of the following converge conditionally.

c conditionally

[✓ I.] $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n+3}}$ alt series converges

$\sum_{n=1}^{\infty} \left| (-1)^n \frac{1}{\sqrt{n+3}} \right|$ diverges p series

II. $\sum_{n=1}^{\infty} (-1)^n \frac{n^2+3n-4}{n!}$ Factorial in denominator converges abs.

c cond.

[✓ III.] $\sum_{n=1}^{\infty} (-1)^n \frac{n-2}{n^2+2}$ alt series (converges)

$\sum_{n=1}^{\infty} \left| (-1)^n \frac{n-2}{n^2+2} \right| \Rightarrow$ like $\frac{1}{n}$ diverges

A. I only ~~B~~. II only ~~C~~. III only [✓ D.] I and III ~~E~~. II and III

$\lim_{n\to\infty} u_n \to 0$

$u_{n+1} < u_n$ for $n \geq N$

$u_n > 0$ for $n \geq N$

32. Find the interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(x-3)^n}{2^n \cdot n}$$

Ratio test

$$\lim_{n\to\infty} \frac{\overset{(x-3)^1}{\cancel{(x-3)^{n+1}}}}{\underset{2^1}{\cancel{2^{n+1}}}\cdot\cancel{(n+1)}} \cdot \frac{\cancel{2^n \cdot n}}{\cancel{(x-3)^n}}$$

$$\left|\frac{x-3}{2}\right| < 1 \quad \text{converge}$$

$$-1 < \frac{x-3}{2} < 1$$

$$-2 < x-3 < 2$$

$$1 < x < 5$$

Test endpoints

$$\sum_{n=1}^{\infty} \frac{(1-3)^n}{2^n \cdot n}$$

$$\frac{\left(\frac{(-2)^n}{2^n}\right)}{\cdot n}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

alt harmonic
converges

$$\sum_{n=1}^{\infty} \frac{(5-3)^n}{2^n \cdot n}$$

$$\frac{\left(\frac{2^n}{2^n}\right)}{n}$$

$$\sum_{n=1}^{\infty} \frac{1^n}{n}$$

harmonic

diverges

interval of convergence $\boxed{[1,5)}$

33. Find the length of the curve.

X = 2sin(t), y = cos(t), $0 \le t \le \pi$

parametric

$$L = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$\frac{dx}{dt} = 2\cos(t)$

$\frac{dy}{dt} = -\sin(t)$

$$\int_0^{\pi} \sqrt{4\cos^2 t + \sin^2 t}\, dt$$

use fnint

≈ 4.8442

34. Let $u = \langle 7, -2 \rangle$ *and* $v = \langle 3, 5 \rangle$.

(a) Find 3**u** – 5**v**

$3\langle 7, -2 \rangle - 5\langle 3, 5 \rangle$

$\langle 21, -6 \rangle - \langle 15, 25 \rangle$

$\langle 21-15, -6-25 \rangle$

$\boxed{\langle 6, {}^{-}31 \rangle}$

(b) Find the magnitude of **u.**

$\sqrt{7^2+(-2)^2}$

$\sqrt{49+4}$

$\boxed{\sqrt{53}}$

35. A particle moves in the plane with position vector
$r(t) = \langle 4\cos(t), \sqrt{2}\sin(t) \rangle$.

Find the velocity vector v(t) and the acceleration vector a(t)

$$r(t) = \langle 4\cos t, \sqrt{2}\sin t \rangle$$

$$\boxed{v(t) = \langle -4\sin t, \sqrt{2}\cos t \rangle}$$

$$\boxed{a(t) = \langle -4\cos t, -\sqrt{2}\sin t \rangle}$$

36. (a) Graph the polar curve $r = 3\cos(2\theta)$

(b) What is the shortest length a θ -interval can have and still produce the graph?

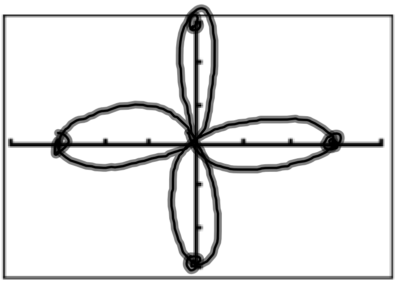
$r = 3\cos(2\theta)$

rose 4 petals

length of petal 3

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
r	3	-3	3	-3	3

36. (a)



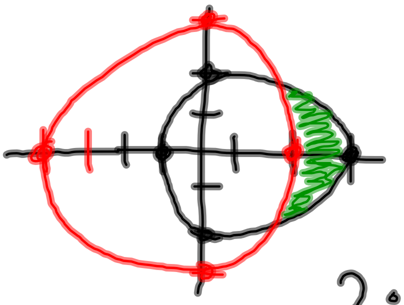
[-4,4] by [-3,3]

36 (b) 2π

n=even takes 2π

n=odd takes 1π

37. Find the area of the region inside the oval limacon $r = 2+ \cos\theta$ and outside the oval limacon $r = 3 - \cos\theta$.



$$\int \frac{1}{2} r_o^2 - \frac{1}{2} r_i^2$$

sym

$$2 \cdot \frac{1}{2} \int_0^{\frac{\pi}{3}} (2+\cos\theta)^2 - (3-\cos\theta)^2 \, d\theta$$

$$\int_0^{\frac{\pi}{3}} 4 + 4\cos\theta + \cos^2\theta - (9 - 6\cos\theta + \cos^2\theta) \, d\theta$$

$$\int_0^{\frac{\pi}{3}} 4 + 4\cos\theta \cancel{+\cos^2\theta} - 9 + 6\cos\theta \cancel{-\cos^2\theta} \, d\theta$$

$$\int_0^{\frac{\pi}{3}} -5 + 10\cos\theta \, d\theta$$

$$-5\theta + 10\sin\theta \Big|_0^{\frac{\pi}{3}}$$

$$\left(-5\left(\frac{\pi}{3}\right) + 10\sin\frac{\pi}{3}\right) - \cancel{(0 + 10\sin 0)}$$

$$-\frac{5\pi}{3} + 10\left(\frac{\sqrt{3}}{2}\right)$$

$$\boxed{-\frac{5\pi}{3} + 5\sqrt{3}} \qquad \approx 3.424$$