The Hyperbola

***Hyperbola*:** The collection of all points in the plane, the difference of whose distances from two fixed points, called the **foci,** is a constant.

***Transverse Axis*:** The line containing the foci.

***Center*:** The midpoint of the line segment joining the foci.

***Conjugate Axis*:** The line through the center and perpendicular to the transverse axis.

***Branches*:** The separate curves of the hyperbola. They are symmetric with respect to the transverse axis, conjugate axis, and center.

***Vertices*:** The points of intersection of the hyperbola and the transverse axis.

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$
$$a^2 + b^2 = c^2$$

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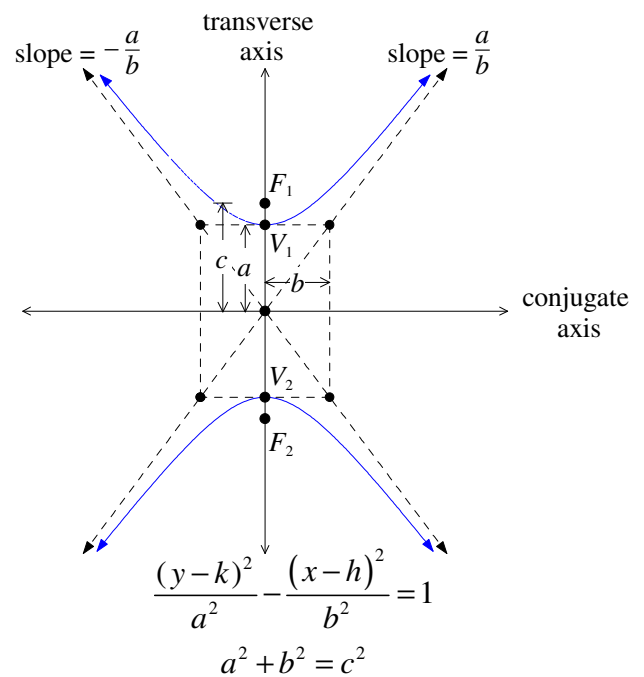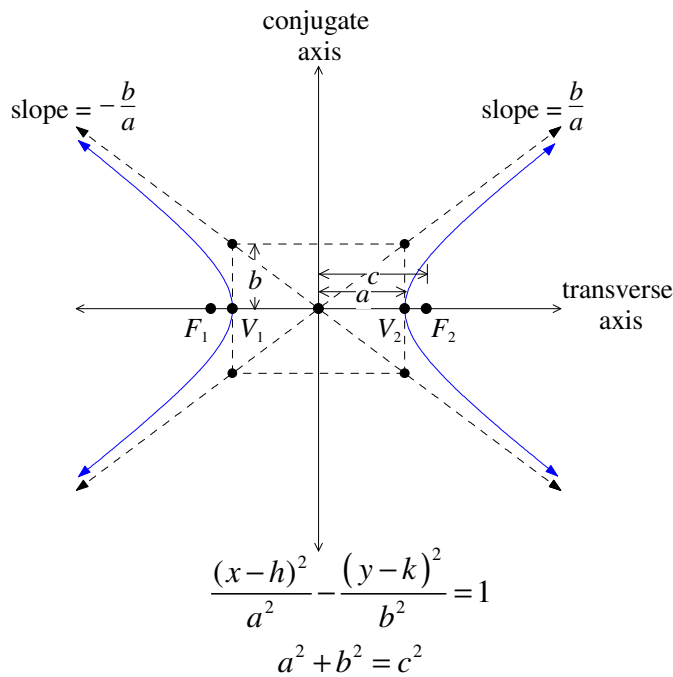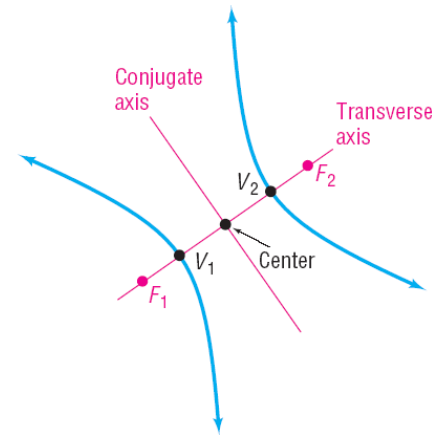
a = distance from center to vertices

c = distance from center to foci

b used to find the width of branches and slope of asymptotes

When finding the equations of the asymptotes, remember that $m = \dfrac{\text{change in } y}{\text{change in } x}$, or, in this case,
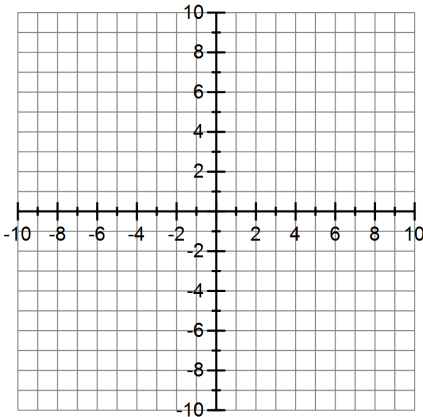
$m = \pm\dfrac{\sqrt{\text{\# under } y^2 \text{ term}}}{\sqrt{\text{\# under } x^2 \text{ term}}}$, then use point slope form $y - y_1 = m(x - x_1)$ with the center (h,k) as (x_1, y_1).

Examples: Find the center, transverse axis, vertices, foci, and asymptotes. Graph each equation.
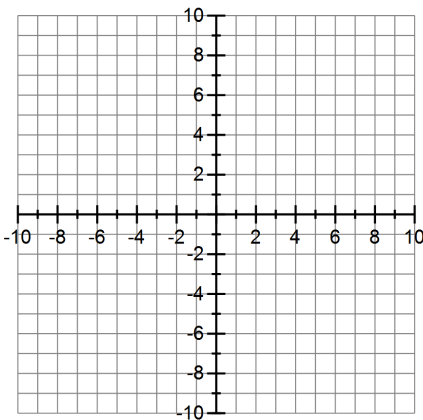
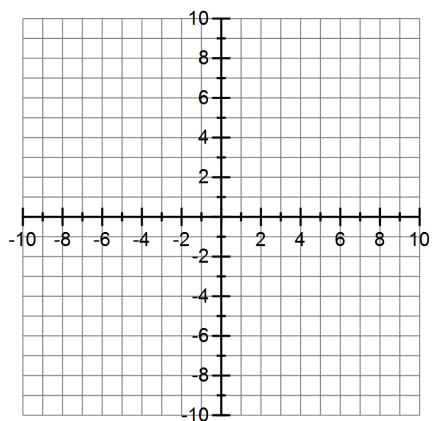a) $\dfrac{y^2}{16} - \dfrac{x^2}{4} = 1$
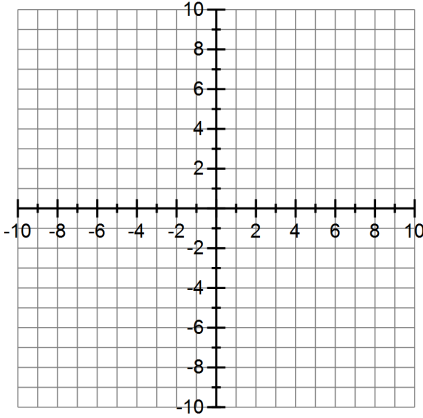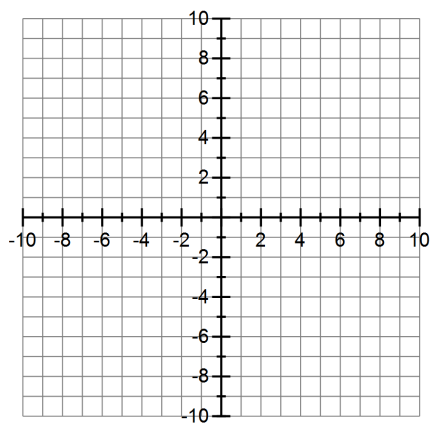

b) $\dfrac{x^2}{9} - \dfrac{y^2}{25} = 1$
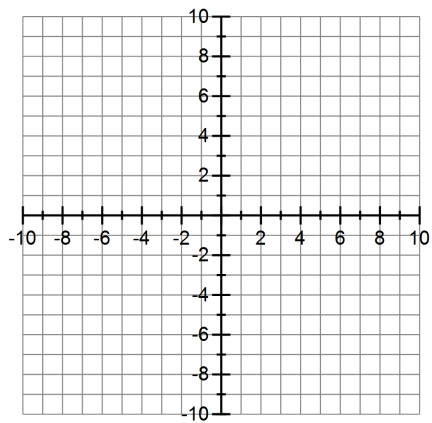

c) $4x^2 - 9y^2 = 36$

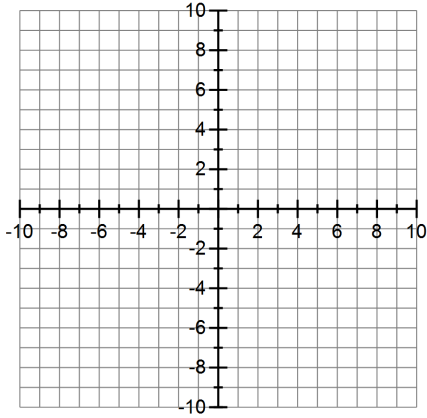d) $\dfrac{(y+3)^2}{4} - \dfrac{(x-2)^2}{9} = 1$

e) $(x+4)^2 - (y-3)^2 = 9$

f) $y^2 - x^2 - 4y + 4x - 1 = 0$

g) $2x^2 - y^2 + 8x + 2y + 3 = 0$

Examples: Write the equation of the hyperbola described.
a) Center at $(1,4)$; Focus at $(-2,4)$; Vertex at $(0,4)$

b) Focus at $(-3,5)$; Vertices at $(-3,4)$ and $(-3,0)$