The Ellipse

Ellipse: The collection of all points in the plane, the sum of whose distances from two fixed points, called the *foci*, F_1 and F_2, is a constant.
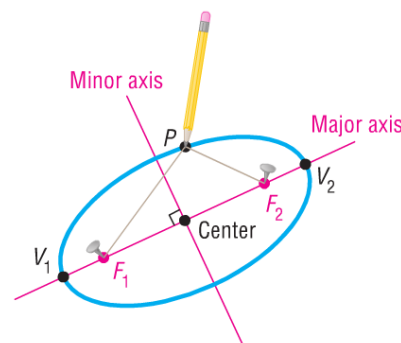
Major Axis: The line containing the foci.
Center: The midpoint of the line segment joining the two foci.
Minor Axis: The line through the center and perpendicular to the major axis.
*Vertices*: The points of intersection of the ellipse and the major axis.
Covertices: The points of intersection of the ellipse and the minor axis.

Standard Form of the Equation of an Ellipse with Center at (h,k)

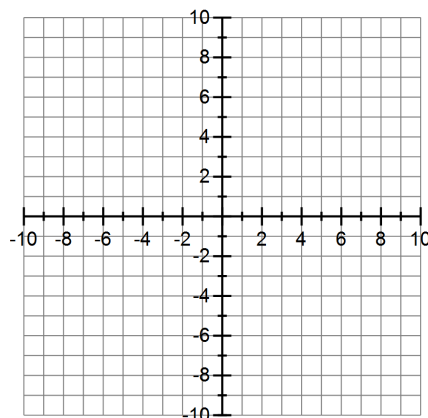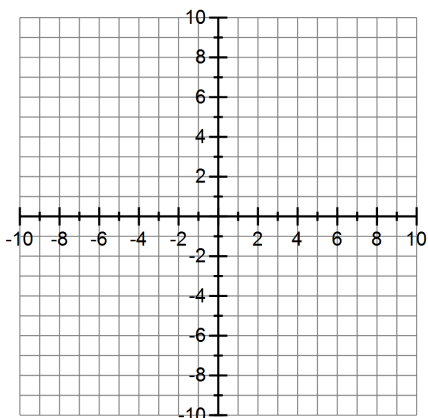| Equation	Description	Picture
$\dfrac{(x-h)^2}{a^2}+\dfrac{(y-k)^2}{b^2}=1$ $a>b>0$ and $a^2-b^2=c^2$	Major axis parallel to x-axis	
$\dfrac{(x-h)^2}{b^2}+\dfrac{(y-k)^2}{a^2}=1$ $a>b>0$ and $a^2-b^2=c^2$	Major axis parallel to y-axis	

a = Distance from center to vertices
b = Distance from center to covertices
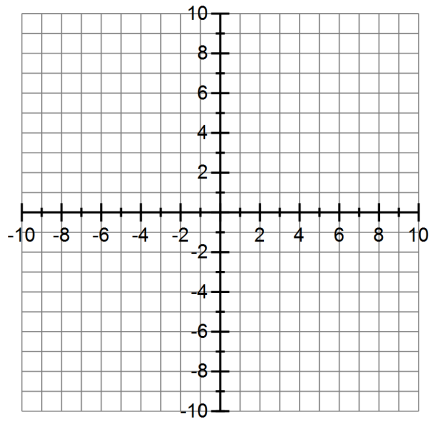**c = Distance from center to foci**
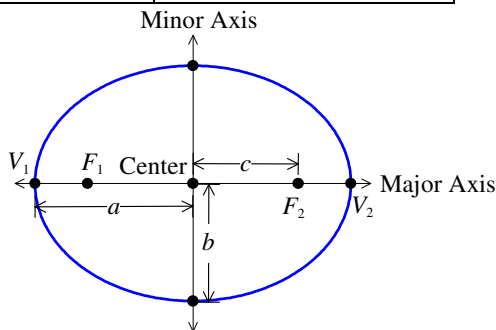
Examples: Find the center, foci, and vertices of each ellipse. Graph each equation.
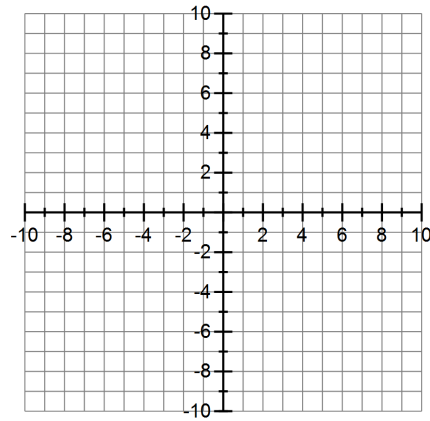
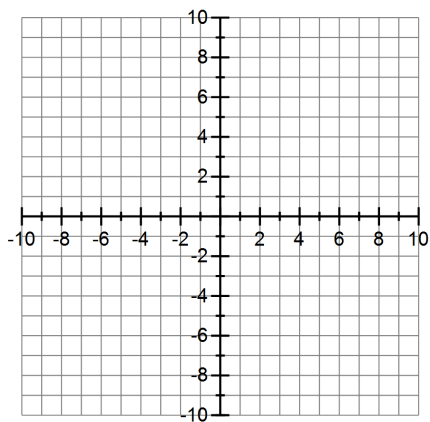a) $\dfrac{x^2}{9}+\dfrac{y^2}{4}=1$

b) $\dfrac{x^2}{16}+\dfrac{y^2}{36}=1$
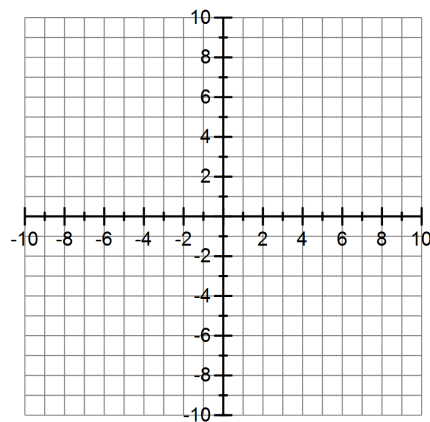
c) $\dfrac{(x+1)^2}{81} + \dfrac{(y-2)^2}{49} = 1$

d) $9(x-3)^2 + (y+2)^2 = 18$

e) $x^2 + 9y^2 + 6x - 18y + 9 = 0$

f) $4x^2 + y^2 + 4y = 0$

Examples: Write the equation of the ellipse having the given characteristics.

a) Foci at $(1,2)$ and $(-3,2)$; Vertex at $(-4,2)$

b) Vertices at $(-1,5)$ and $(-1,-3)$; $c = 1$

c) Center at $(1,2)$; Focus at $(1,4)$; Contains $(2,2)$

Example: A hall 100 feet in length is to be designed as a whispering gallery. If the foci are located 25 feet from the center, how high will the ceiling be at the center?

Example: A bridge is to be built in the shape of a semielliptical arch and is to have a span of 100 feet. The height of the arch, at a distance of 40 feet from the center is to be 10 feet. Find the height of the arch at its center.