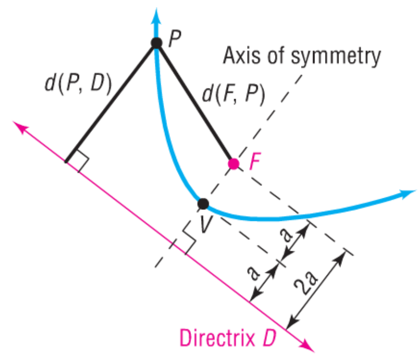
The Parabola



Parabola**:** The collection of all points P in the plane that are the same distance from a fixed point F, called the ***focus*** of the parabola, as they are from a fixed line D, called the ***directrix*** of the parabola.

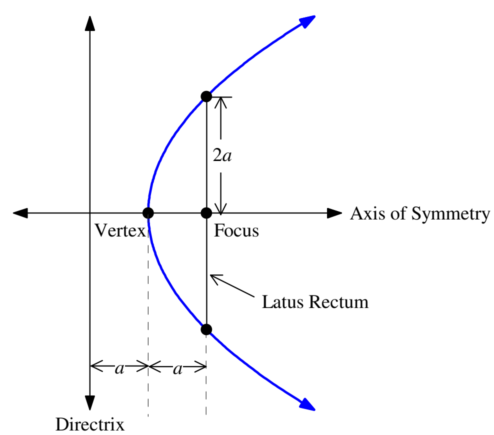
Axis of Symmetry**:** The line through the focus F and perpendicular to the directrix D.

Vertex**:** The point of intersection of the parabola with its axis of symmetry.

General Forms of the Equation of a Parabola with Vertex (h,k)

***a* = Distance from Focus to Vertex** ***a* = Distance from Vertex to Directrix**

Equation	Description	Picture
$(y-k)^2 = 4a(x-h)$	Opens Right, Axis of Symmetry parallel to x-axis	
$(y-k)^2 = -4a(x-h)$	Opens Left, Axis of Symmetry parallel to x-axis	
$(x-h)^2 = 4a(y-k)$	Opens Up, Axis of Symmetry parallel to y-axis	
$(x-h)^2 = -4a(y-k)$	Opens Down, Axis of Symmetry parallel to y-axis	

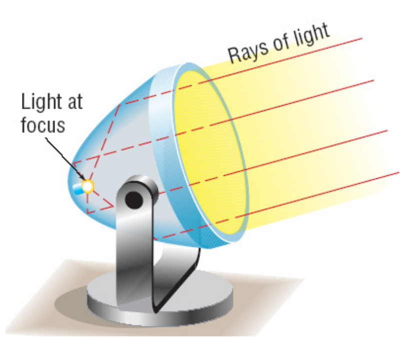


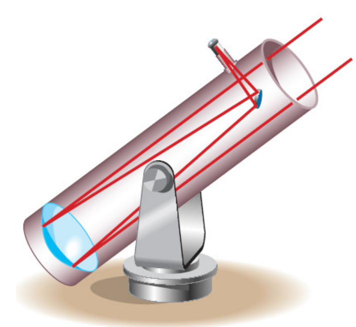
Latus Rectum**:** The line segment with endpoints on the parabola that passes through the focus and is perpendicular to the axis of symmetry. Each of the endpoints is at a distance of $2a$ from the focus.

Paraboloid of Revolution**:** A surface formed by rotating a parabola about its axis of symmetry.

Suppose a mirror is shaped like a paraboloid of revolution. If a light (or other radiation source) is placed at the focus of the parabola, all the rays emanating from the light will reflect off the mirror in lines parallel to the axis of symmetry.

Conversely, when rays of light from a distant source strike the surface of a parabolic mirror, they are reflected to a single point at the focus. This fact is used in the design of telescopes and other optical devices.





Examples: Graph the following parabolas. State the vertex, focus, axis of symmetry, directrix, length of latus rectum, and direction of opening.

a) $x^2 = 8y$ opens up

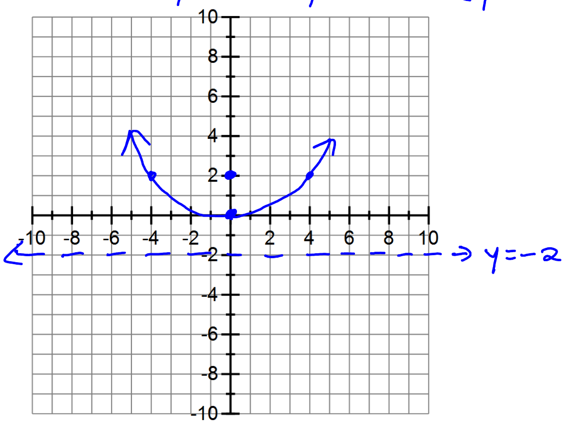
Vertex: $(0,0)$ $4a = 8$

focus: $(0,2)$ $a = 2$

directrix: $y = -2$

length of latus rectum $= 8$ (4 to right of focus, 4 to left)

axis of symmetry: $x = 0$ (y-axis)



b) $y^2 = -16x$ opens left

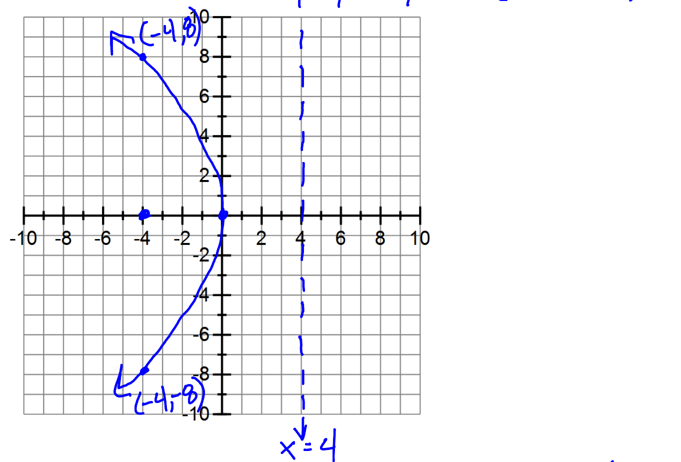
vertex: $(0,0)$ $4a = 16$

focus: $(-4,0)$ $a = 4$

directrix: $x = 4$

length of latus rectum $= 16$ (8 up from focus & 8 down)

axis of symmetry: $y = 0$ (x-axis)



c) $(x-2)^2 = -12(y+1)$ opens down

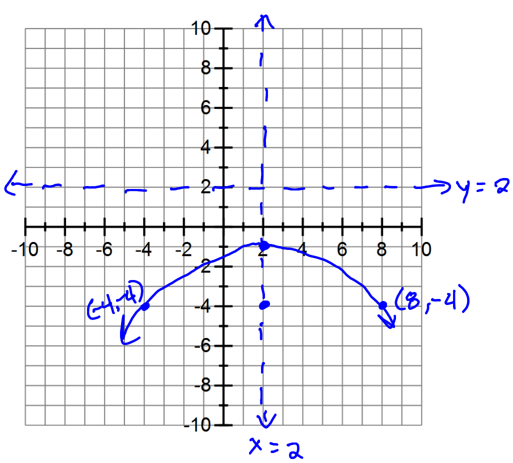
vertex: $(2,-1)$ $4a = 12$

focus: $(2,-4)$ $a = 3$

directrix: $y = 2$

length of latus rectum: 12 (6 left of focus & 6 right)

axis of symmetry: $x = 2$



d) $(y+3)^2 = 2(x-4)$ opens right

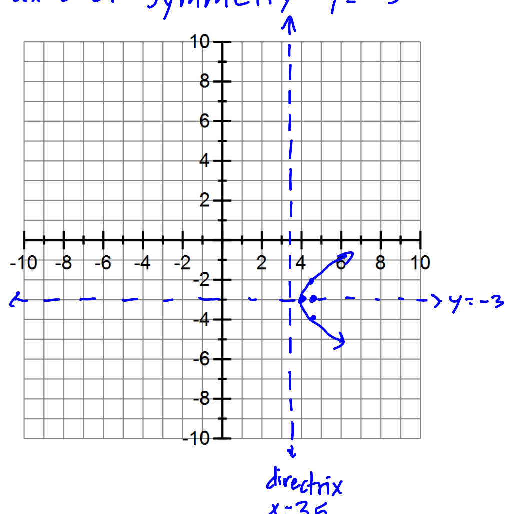
vertex: $(4,-3)$ $4a = 2$

focus: $(4.5,-3)$ $a = \frac{1}{2}$

directrix: $x = 3.5$

length of latus rectum $= 2$ (1 up from focus & 1 down)

axis of symmetry: $y = -3$



Examples: Write each equation in standard form.

a) $y = x^2 + 2x + 2$
$-2 \qquad\qquad -2$

x's on one side, everything else on other side: $x^2 + 2x = y - 2$

complete the square: $\frac{2}{2} = 1$, $1^2 = 1$ $\quad x^2 + 2x + 1 = y - 2 + 1$

write as perfect square: $\boxed{(x+1)^2 = y - 1}$ $\leftarrow (4a = 1)$

If there were anything in front of the y, I would factor it out

b) $\cancel{x} + y^2 = \cancel{6}y - 3$
$-x \quad -6y \quad -6y \quad -x$

$y^2 - 6y = -x - 3$

$y^2 - 6y + 9 = -x - 3 + 9$ $\qquad -\frac{6}{2} = -3$, $(-3)^2 = 9$

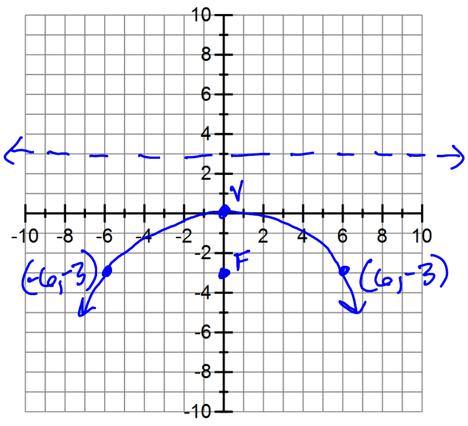
$(y-3)^2 = -x + 6$

Factor out negative

$\boxed{(y-3)^2 = -(x-6)}$

Examples: Find the equation of the parabola described. Find the two points that define the latus rectum and graph the equation.

a) Vertex: $(0,0)$; Focus: $(0,-3)$



opens down

$a = 3$

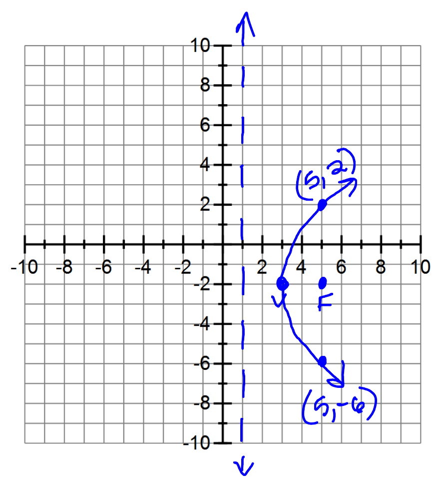
standard form: $x^2 = -4ay$

vertex is (0,0) so x & y are alone

$$\boxed{x^2 = -12y}$$

6 right & 6 left of focus

b) Vertex: $(3,-2)$; Focus: $(5,-2)$



opens right

$a = 2$

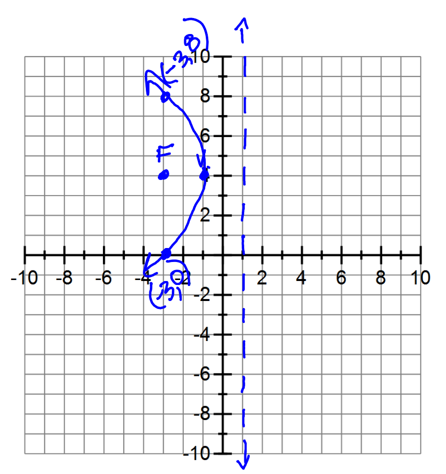
$(h,k) = (3,-2)$

standard form: $(y-k)^2 = 4a(x-h)$

$$\boxed{(y+2)^2 = 8(x-3)}$$

4 up & 4 down from focus

c) Vertex: $(-1,4)$; Directrix: $x = 1$



opens left

$a = 2$

$(h,k) = (-1,4)$

standard form: $(y-k)^2 = -4a(x-h)$

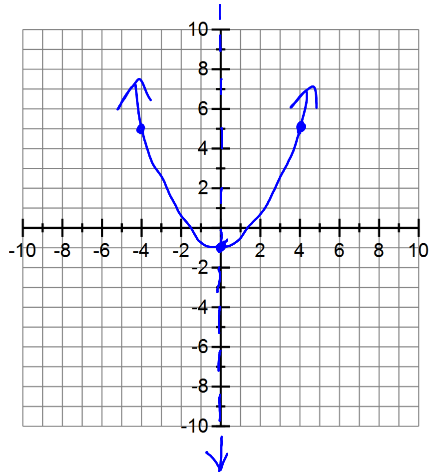
$$\boxed{(y-4)^2 = -8(x+1)}$$

4 up & 4 down from focus

d) Vertex: $(0,-1)$; Axis of Symmetry: y-axis; Contains the point $(4,5)$

Do not assume this is by the focus

(Don't assume $a=5$)



$(x-h)^2 = 4a(y-k)$

Plug in vertex: $x^2 = 4a(y+1)$

Plug in other point for (x,y): $4^2 = 4a(5+1)$

Solve for a: $16 = 4(6)a$

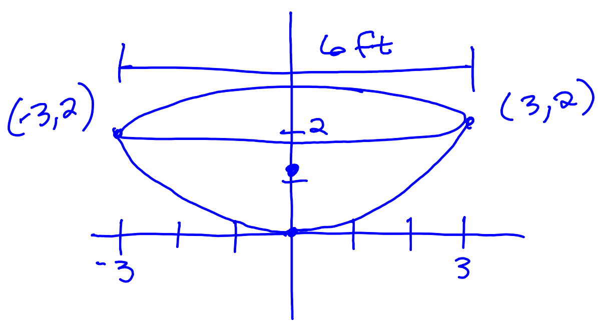
$16 = 24a$

$a = \frac{16}{24} = \frac{2}{3}$

$x^2 = 4\left(\frac{2}{3}\right)(y+1)$

$\boxed{x^2 = \frac{8}{3}(y+1)}$

Example: A cable TV receiving dish is in the shape of a paraboloid of revolution. Find the location of the receiver, which is placed at the focus, if the dish is 6 feet across its opening and 2 feet deep.



Find a

$x^2 = 4ay$

vertex @ $(0,0)$ so x & y are alone

$3^2 = 4a(2)$

$9 = 8a$

$a = \frac{9}{8}$ ft

$\boxed{a = 1.125 \text{ ft}}$