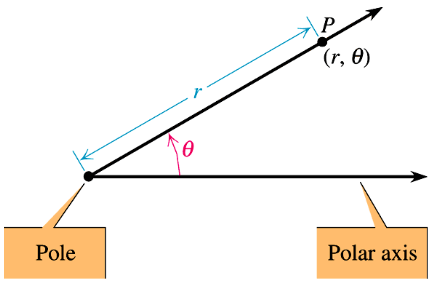
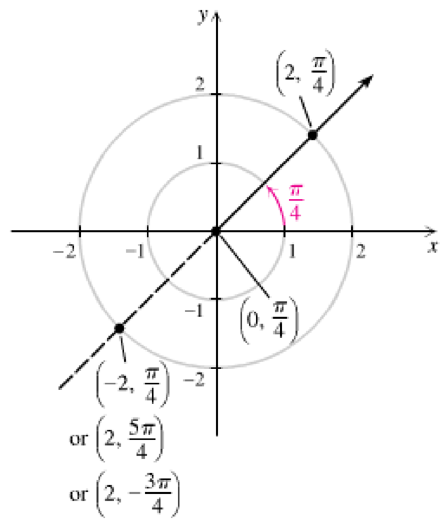
## Polar Coordinates and Polar Equations



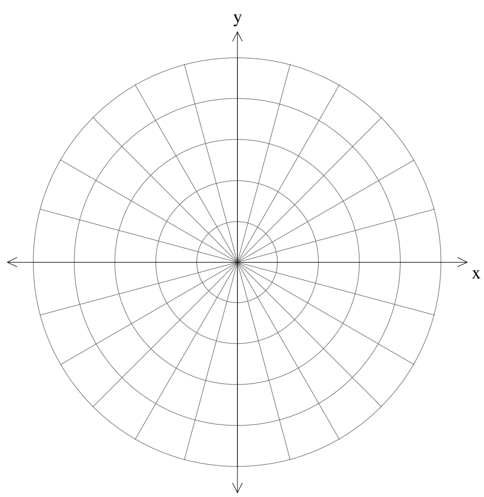

The polar coordinate system is based on a fixed point called the **pole** and a fixed axis called the **polar axis.** Points are represented by ordered pairs in the form $(r,\theta)$, where $r$ is the **directed distance** from the pole and $\theta$ is an angle whose initial side is the polar axis and whose terminal side contains the point.
Typically, we choose the origin as the pole and the positive $x$-axis as the polar axis.




*To graph $(-r,\theta)$, you move in the opposite direction you would move to graph $(r,\theta)$.

Polar coordinates are not unique. The points $\left(-2,\frac{\pi}{4}\right)$, $\left(2,\frac{5\pi}{4}\right)$, and $\left(2,-\frac{3\pi}{4}\right)$ all name the same point.

**Examples:** Plot the points whose polar coordinates are given.
$A\left(3,\frac{\pi}{3}\right)$, $B\left(-1,\frac{\pi}{6}\right)$, $C\left(2,-\frac{7\pi}{4}\right)$, $D\left(-5,-\frac{3\pi}{4}\right)$, $E\left(4,\frac{\pi}{2}\right)$, $F\left(-3,\frac{2\pi}{3}\right)$



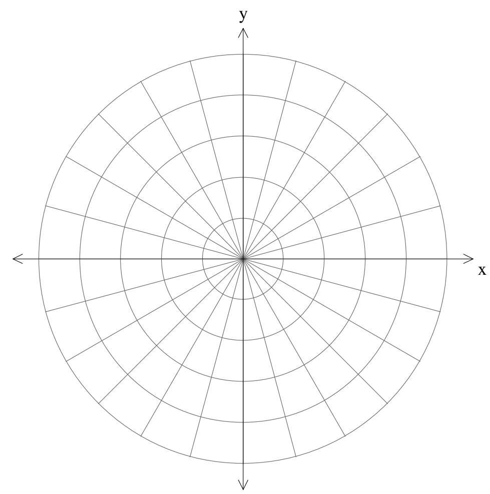

**Polar-Rectangular Conversion Rules**
- To convert $(r,\theta)$ to rectangular coordinates $(x,y)$, use $x = r\cos\theta$ and $y = r\sin\theta$.
- To convert $(x,y)$ to polar coordinates $(r,\theta)$, use $r=\sqrt{x^2+y^2}$ and any angle $\theta$ in standard position whose terminal side contains $(x,y)$.

**Examples:**

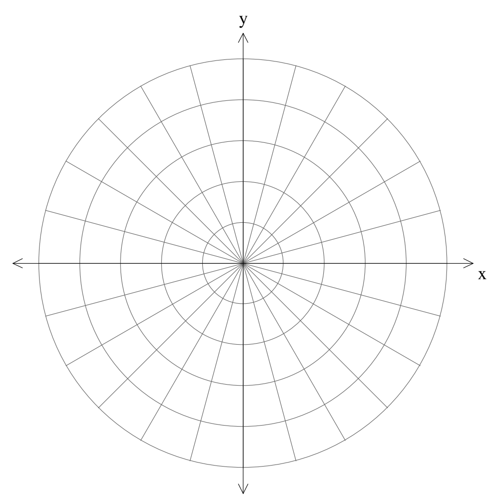
a) Convert $(3,45^\circ)$ to rectangular coordinates.

b) Convert $\left(-2,2\sqrt{3}\right)$ to polar coordinates.

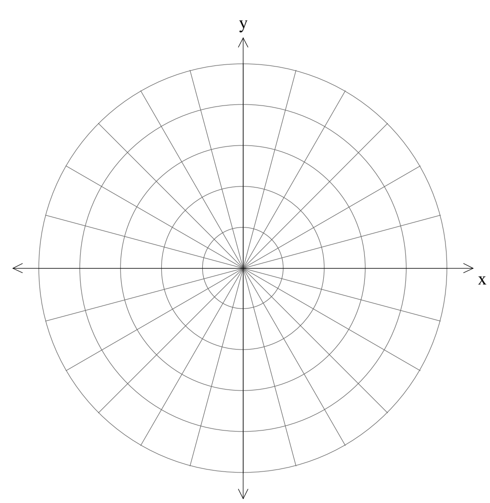
**Graphing Polar Equations**
**Examples:** Sketch the graphs of the following:
a) $r = 4\sin\theta$



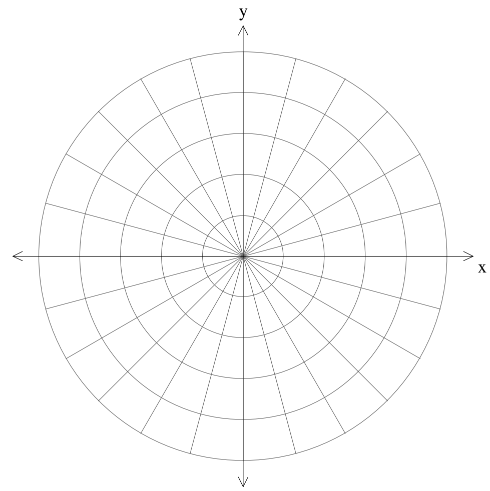

b) $r = 3\cos(2\theta)$



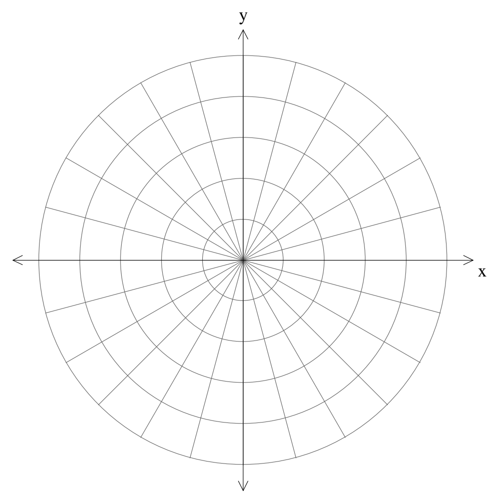

c) $r = 2 + 3\sin\theta$



d) $r^2 = 4\sin(2\theta)$




e) $r = 5\sin(4\theta)$




**Examples:** Convert equations from polar to rectangular form.

a) Convert $r = 3\sin\theta$ to a rectangular equation.

b) Convert $r = \dfrac{4}{1+\sin\theta}$ to a rectangular equation.

c) Convert $r = 5\sec\theta$ to a rectangular equation.

d) Convert $r = 5$ to a rectangular equation.

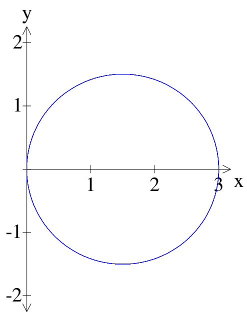
**Examples:** Convert equations from rectangular to polar form.
a) Convert $y = 7$ to a polar equation.

b) Convert $y = -2x + 5$ to a polar equation.

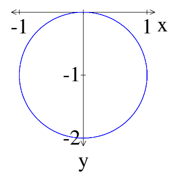
c) Convert $x^2 + (y-1)^2 = 1$ to a polar equation.

1) Lines through the origin are of the form $\theta = \alpha$.
Vertical lines are of the form $r = a\sec\theta$.
Horizontal lines are of the form $r = a\csc\theta$.

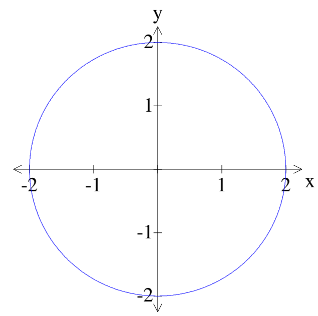
2) Circles come in three forms: $r = a\cos\theta$, $r = a\sin\theta$, and $r = a$.




$r = 3\cos\theta$

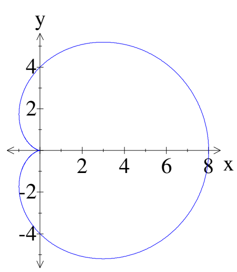



$r = -2\sin\theta$

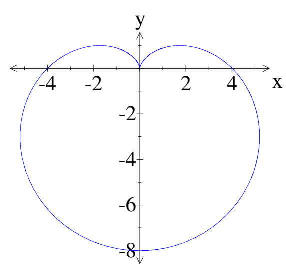



$r = 2$

3) Cardioids have the form $r = a \pm a\cos\theta$ or $r = a \pm a\sin\theta$.
Cardioids pass through the pole.

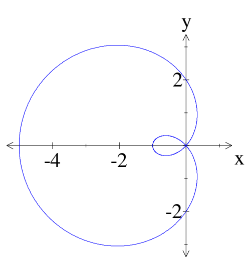



$r = 4 + 4\cos\theta$

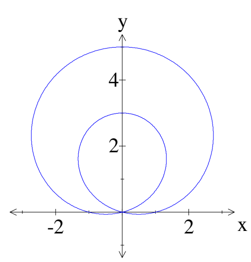



$r = 4 - 4\sin\theta$

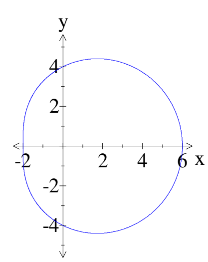
4) Limaçons have the form $r = a \pm b\cos\theta$ or $r = a \pm b\sin\theta$.
Limaçons have an inner loop if $0 < a < b$ and have no inner loop if $0 < b < a$.
The graph of a limaçon with an inner loop passes through the pole twice.
The graph of a limaçon with no inner loop does not pass through the pole.



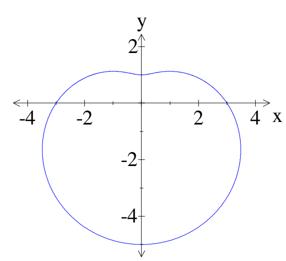

$r = 2 - 3\cos\theta$




$r = 1 + 4\sin\theta$

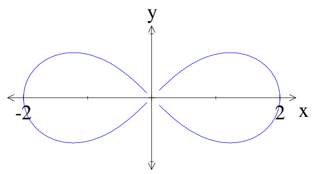



$r = 4 + 2\cos\theta$

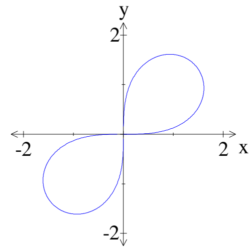



$r = 3 - 2\sin\theta$

5) Lemniscates have the form $r^2 = a^2 \cos(2\theta)$ or $r^2 = a^2 \sin(2\theta)$.

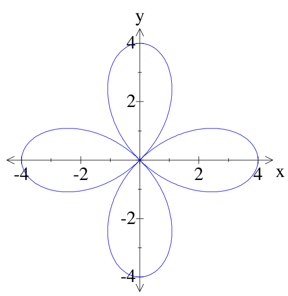



$r^2 = 4\cos(2\theta)$

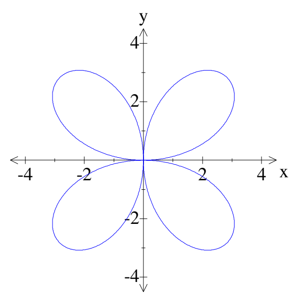



$r^2 = 4\sin(2\theta)$

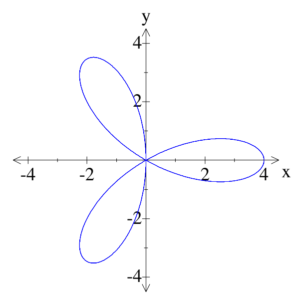
6) Roses have the form $r = a\cos(n\theta) + b$ and $r = a\sin(n\theta) + b$.
If $n$ is even, there are $2n$ loops in the rose.
If $n$ is odd, there are $n$ loops in the rose.



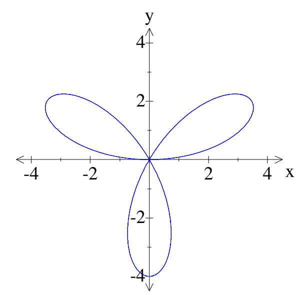

$r = 4\cos(2\theta)$



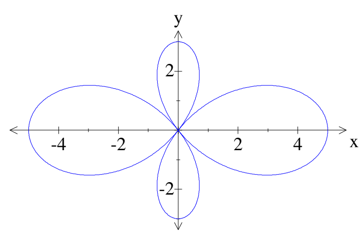

$r = 4\sin(2\theta)$



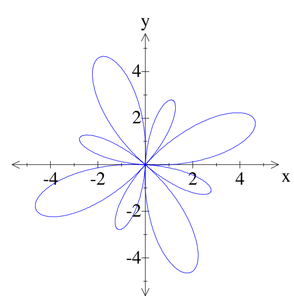

$r = 4\cos(3\theta)$




$r = 4\sin(3\theta)$




$r = 4\cos(2\theta) + 1$




$r = 4\sin(4\theta) + 1$