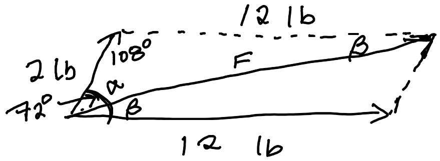
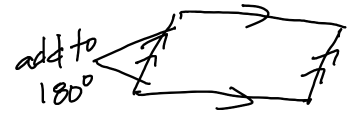
Applications of Vectors

Example: Forces of 2 lb and 12 lb are acting at an angle of 72° to each other. Find the magnitude of the resultant force and the angle between the resultant and each force.



$|\vec{F}|^2 = 2^2 + 12^2 - 2(2)(12)\cos 108°$

$|\vec{F}| = 12.8$ lb



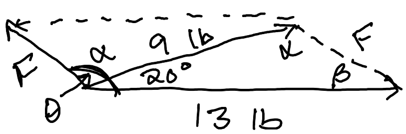
$\frac{\sin 108°}{12.8} = \frac{\sin\alpha}{12}$

$\sin\alpha = \frac{12\sin 108°}{12.8} \rightarrow \alpha \approx 63.4°$

$\beta = 72° - 63.4°$

$\beta \approx 8.6°$

Example: The resultant of a 13-lb force and another force has a magnitude of 9 lb at an angle of 20° with the 13-lb force. Find the magnitude of the other force and the angle between the two forces.



$|\vec{F}|^2 = 9^2 + 13^2 - 2(9)(13)\cos 20°$

$|\vec{F}| = 5.5$ lb

$\frac{\sin 20°}{5.5} = \frac{\sin\beta}{9}$

$\beta \approx 34.1°$

α might be obtuse, so don't want to use Law of Sines to get α.

$\alpha \approx 180° - (20° + 34.1°)$

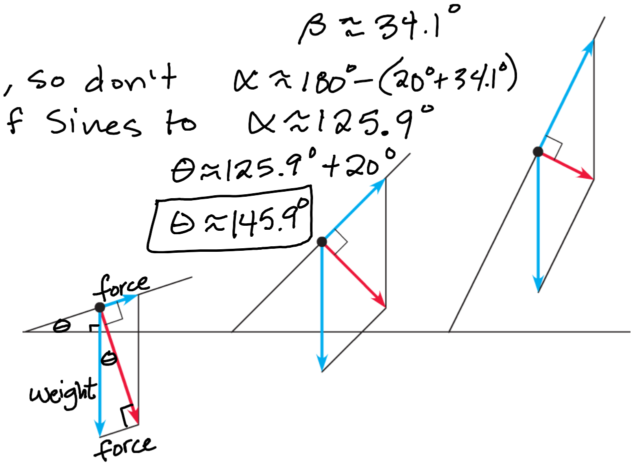
$\alpha \approx 125.9°$

$\theta \approx 125.9° + 20°$

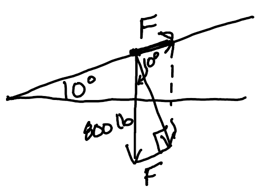
$\theta \approx 145.9°$

Inclined Plane Problems: The weight of an object is always modeled as a vertical vector and the force required to move the object is modeled as a vector parallel to the inclined plane. Its length increases as the incline increases. The resultant of these two forces is a vector perpendicular to the inclined plane. It is what a bathroom scale would read if trapped between the object and the plane.

Always SOH-CAH-TOA



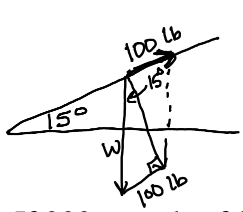
Example: Find the amount of force required to push an 800-pound block of ice up a ramp that is inclined 10°.



$\sin 10° = \frac{|\vec{F}|}{800}$

$|\vec{F}| = 800\sin 10° \approx 138.9$ lb

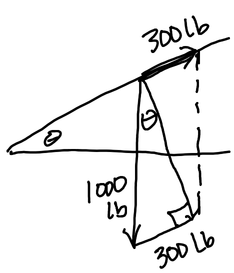
Example: A landscaper uses 100 pounds of force to pull a cart full of rocks up a driveway that is inclined 15°. What is the weight of the cart?



$\sin 15° = \frac{100}{|\vec{w}|}$

$|\vec{w}| = \frac{100}{\sin 15°} \approx 386.4$ lb

Example: If 300 pounds of force is required to push a 1000-pound safe up a ramp, then what is the angle of inclination of the ramp?

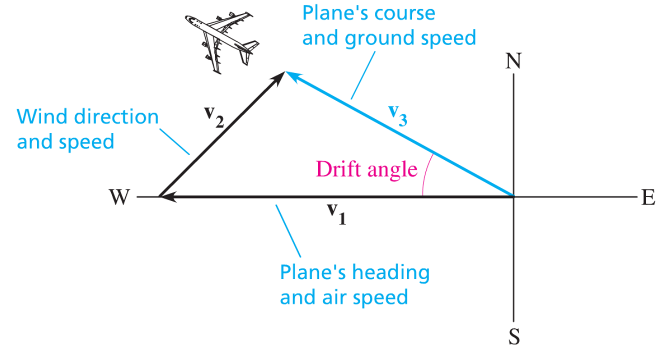


$\sin\theta = \frac{300}{1000}$

$\theta = \sin^{-1}\left(\frac{300}{1000}\right)$

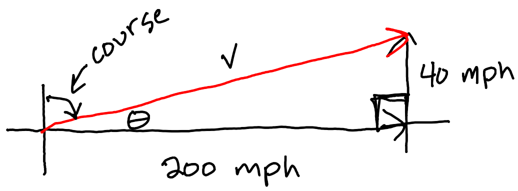
$\theta \approx 17.5°$

Navigation Problems: Wind affects the speed and direction of a plane. The **heading** and **air speed** are the direction and speed of the plane before wind is taken into account. The **course** and **ground speed** are the direction and speed of the plane with wind taken into account. The angle between the heading and the course is the **drift angle**.



Example: An airplane is headed due east with an air speed of 200 mph. The wind is out of the south (bearing 0°) at 40 mph. Find the ground speed, the drift angle, and the course of the airplane.

↑ blowing north



right triangle!

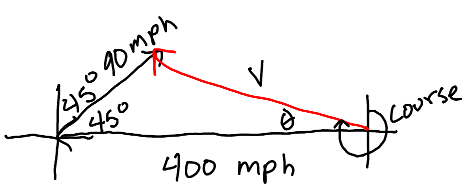
$|\vec{v}| = \sqrt{200^2 + 40^2} \approx \boxed{204.0 \text{ mph}}$ ← ground speed

$\tan\theta = \frac{40}{200}$ $\theta = \tan^{-1}\left(\frac{40}{200}\right) \approx \boxed{11.3°}$ ← drift angle

$\text{Course} = 90° - 11.3° \approx \boxed{78.7°}$ ← course (bearing of plane)

Example: An airplane is headed due west with an air speed of 400 mph. The wind is out of the southwest (bearing 45°) at 90 mph. Find the ground speed, the drift angle, and the course of the airplane.

blowing NE



not a right Δ

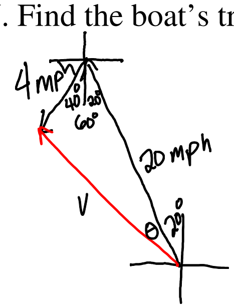
$|\vec{v}|^2 = 400^2 + 90^2 - 2(400)(90)\cos 45°$

$|\vec{v}| \approx \boxed{342.3 \text{ mph}}$ ← ground speed

$\frac{\sin 45°}{342.3} = \frac{\sin\theta}{90} \rightarrow \theta \approx \boxed{10.7°}$ ← drift angle

$\text{Course} \approx 270° + 10.7° \approx \boxed{280.7°}$ ← course

Example: A boat is traveling at 20 mph with a bearing of N20°W. The current is moving at 4mph with a bearing of S40°W. Find the boat's true speed, drift angle, and course.



$|\vec{v}|^2 = 4^2 + 20^2 - 2(4)(20)\cos 60° = 336$

$|\vec{v}| \approx \boxed{18.3 \text{ mph}}$ ← true speed

$\frac{\sin 60°}{18.3} = \frac{\sin\theta}{4} \rightarrow \theta \approx \boxed{10.9°}$ ← drift angle

$20° + 10.9° \approx 30.9°$

course: $\boxed{N30.9°W}$