Vectors

Scalar Quantities: Quantities such as length, area, volume, temperature, and time, which have magnitude (size), but no direction.

Vector Quantities: Quantities that involve both a magnitude and direction, such as velocity, acceleration, and force. These quantities can be represented by **directed line segments** called **vectors**.

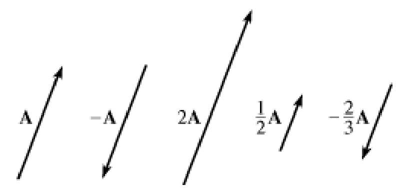
The length of a vector represents the **magnitude** of the vector quantity. The **direction** is indicated by the position of the vector and the arrowhead at one end.

Notation: $\overrightarrow{AB}$ is used to name a vector with **initial point** *A* and **terminal point** *B*. Vectors may also be denoted by bold letters. $\overrightarrow{AB}$ can also be written as **AB**. If the initial and terminal points are not specified, vectors can be named by a single uppercase or lowercase letter (eg. $\vec{b}$, $\vec{B}$, **b**, or **B**). The magnitude of vector **A** is written $|\mathbf{A}|$.

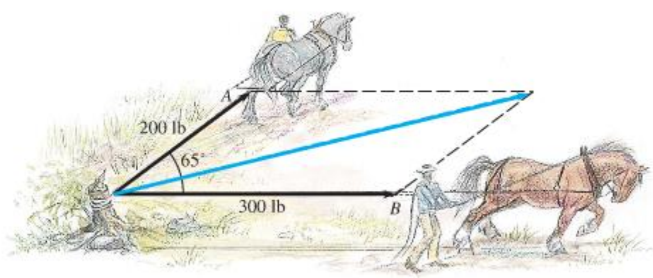
Equal Vectors: Vectors with the same magnitude and direction. They do not have to be in the same place.

Zero Vector: A vector with no magnitude and no direction. It is denoted by $\mathbf{0}$.

Scalar Multiplication: For any scalar *k* and vector **A,** *k***A** is a vector with magnitude $|k|$ times the magnitude of **A**. If $k > 0$, then the direction of $k\mathbf{A}$ is the same as the direction of **A**. If $k < 0$, the direction of $k\mathbf{A}$ is opposite to the direction of **A**. If $k = 0$, then $k\mathbf{A} = \mathbf{0}$.

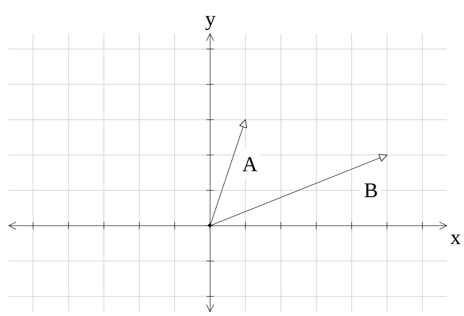


Suppose two draft horses are pulling on a tree stump with forces of 200 pounds and 300 pounds, with an angle of 65° between the forces. If **A** and **B** had the same direction, then there would be a total force of 500 pounds acting on the stump, but the total force is less because of the angle between the forces. By the **parallelogram law**, the force acting along the diagonal of the parallelogram, with a magnitude equal to the length of the diagonal, has the same effect on the stump as the two forces **A** and **B**. The force **A** + **B** acting along the diagonal is called the **sum** or **resultant** of **A** and **B**.

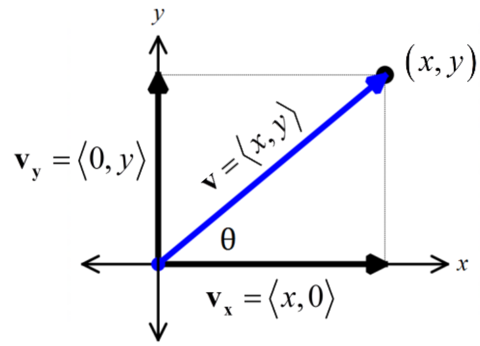


Vector Addition: To find the resultant or sum **A** + **B** of any vectors **A** and **B**, draw **B** so that the initial point of **B** is at the terminal point of **A**. The vector that begins at the initial point of **A** and ends at the terminal point of **B** is the vector **A** + **B**. For every vector **B**, there is a vector **– B**, with the same magnitude as **B**, but the opposite direction. **A – B = A +** (**– B**).

Example: Sketch the vectors **A** + **B** and **A** – **B**.



Any nonzero vector **v** is the sum of a **horizontal component**, $\mathbf{v_x}$, and a **vertical component**, $\mathbf{v_y}$. If a vector **v** is placed in a rectangular coordinate system so that its initial point is the origin, then **v** is called a **position vector**. The angle θ formed by the positive x-axis and a position vector is the **direction angle** for the position vector.



Component Form: The notation $\langle x, y \rangle$ is used to define a position vector with terminal point (x, y). This is called component form because the horizontal component is $\mathbf{v_x} = \langle x, 0 \rangle$ and its vertical component is $\mathbf{v_y} = \langle 0, y \rangle$.

The magnitude of the vector $\mathbf{v} = \langle x, y \rangle$ is $|\mathbf{v}| = r = \sqrt{x^2 + y^2}$. To find the direction angle, use $\sin\theta = \dfrac{y}{r}$, $\cos\theta = \dfrac{x}{r}$, and $\tan\theta = \dfrac{y}{x}$.

If a vector has magnitude r and direction angle θ, its component form is $\langle r\cos\theta,\ r\sin\theta \rangle$.

Examples: Find the component form for each vector **v** with the given magnitude and direction angle θ. Round to the nearest tenth if necessary.

a) $|\mathbf{v}| = 12,\ \theta = 45°$ b) $|\mathbf{v}| = 50,\ \theta = 120°$ c) $|\mathbf{v}| = 445,\ \theta = 211.1°$

Examples: Find the magnitude and direction angle of each vector.

a) $\mathbf{v} = \langle 1, -1 \rangle$ b) $\mathbf{v} = \langle -2, 2\sqrt{3} \rangle$ c) $\mathbf{v} = \langle -4, -5 \rangle$

If $\mathbf{A} = \langle a_1, a_2 \rangle$, $\mathbf{B} = \langle b_1, b_2 \rangle$, and k is a scalar, then

Vector Arithmetic:

1.	$k\mathbf{A} = \langle ka_1, ka_2 \rangle$	**Scalar Product**
2.	$\mathbf{A} + \mathbf{B} = \langle a_1 + b_1, a_2 + b_2 \rangle$	**Vector Sum**
3.	$\mathbf{A} - \mathbf{B} = \langle a_1 - b_1, a_2 - b_2 \rangle$	**Vector Difference**
4.	$\mathbf{A} \cdot \mathbf{B} = a_1 b_1 + a_2 b_2$	**Dot Product**

Examples: Let $\mathbf{w} = \langle -1, -3 \rangle$ and $\mathbf{v} = \langle -3, 4 \rangle$. Perform the operations indicated.

a) $\mathbf{w} - \mathbf{v}$ b) $-8\mathbf{v}$ c) $3\mathbf{w} + 4\mathbf{v}$ d) $\mathbf{w} \cdot \mathbf{v}$

The Angle Between Two Vectors:

If **A** and **B** are nonzero vectors and α is the smallest positive angle between them, then $\alpha = \cos^{-1}\left(\dfrac{\mathbf{A} \cdot \mathbf{B}}{|\mathbf{A}||\mathbf{B}|} \right)$.

Examples: Find the smallest positive angle between the following vectors:

a) $\langle 1, 3 \rangle$ and $\langle 5, 2 \rangle$ b) $\langle -5, 1 \rangle$ and $\langle 7, 3 \rangle$

Parallel Vectors: The vectors **A** and **B** are parallel if and only if $\mathbf{A} = k\mathbf{B}$ for a nonzero scalar k.
Perpendicular Vectors: The vectors **A** and **B** are perpendicular if and only if $\mathbf{A} \cdot \mathbf{B} = 0$.

Examples: Determine whether each pair of vectors is parallel, perpendicular, or neither.

a) $\langle -2, 3 \rangle$ and $\langle 6, 4 \rangle$ b) $\langle 2, -5 \rangle$ and $\langle -4, 10 \rangle$ c) $\langle 2, 6 \rangle$ and $\langle 6, 2 \rangle$

The vectors $\mathbf{i} = \langle 1,0 \rangle$ and $\mathbf{j} = \langle 0,1 \rangle$ are called **unit vectors** because each has magnitude one. For any vector $\langle a_1, a_2 \rangle$, we have $\langle a_1, a_2 \rangle = a_1 \langle 1,0 \rangle + a_2 \langle 0,1 \rangle = a_1\mathbf{i} + a_2\mathbf{j}.$ The form $a_1\mathbf{i} + a_2\mathbf{j}$ is called a **linear combination** of the vectors **i** and **j**.

Examples: Write each vector as a linear combination of the unit vectors **i** and **j**.

a) $\mathbf{A} = \langle 2,3 \rangle$

b) $\mathbf{B} = \langle -1,7 \rangle$

c) $\mathbf{C} = \langle 0,-9 \rangle$