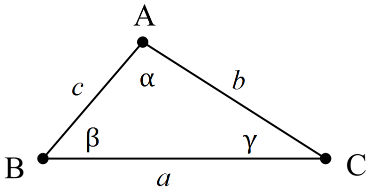
Precalculus – 6.4 Notes

The Law of Sines

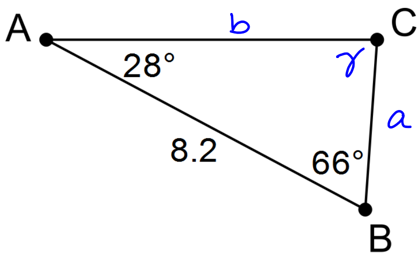
Solving a triangle means finding the measures of all the sides and angles. An **oblique triangle** is a triangle without a right angle. To solve an oblique triangle, we must know three pieces of information, at least one of which must be the length of a side. (Three angles define an infinite number of triangles).



The Law of Sines: In any triangle, $\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$

ASA or AAS: Find the third angle using the fact that the three angles of a triangle add to 180°. Then use the law of sines to find the other two sides of the triangle.

Example: $\alpha = 28^\circ$, $\beta = 66^\circ$, $c = 8.2$



$\gamma = 180^\circ - (28^\circ + 66^\circ) = \boxed{86^\circ}$

$$\frac{\sin 86^\circ}{8.2} = \frac{\sin 28^\circ}{a}$$

$a \sin 86^\circ = 8.2 \sin 28^\circ$

$$a = \frac{8.2 \sin 28^\circ}{\sin 86^\circ} = \boxed{3.9}$$

$$\frac{\sin 86^\circ}{8.2} = \frac{\sin 66^\circ}{b}$$

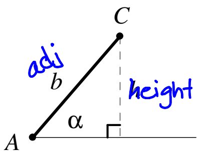
$8.2 \sin 66^\circ = b \sin 86^\circ$

$$b = \frac{8.2 \sin 66^\circ}{\sin 86^\circ}$$

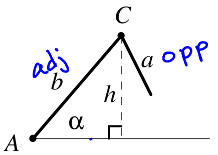
$\boxed{b = 7.5}$

SSA (The Ambiguous Case): If you know two sides and a non-included angle (an angle that is not between the sides), there may be zero, one, or two possible triangles that fit the given measurements. To figure out how many triangles there are for an *acute* angle *α*, do the following:

1. Draw the given angle (α) in standard position with a terminal side of length b. Don't draw side a yet.
2. Let h be an altitude from C to the initial side of α.



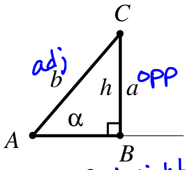
3. Since $\sin\alpha = h/b$, $h = b\sin\alpha$. Compare h to a as follows:
   a. If $a < h$, then no triangle can be formed. opp < height
   b. If $a = h$, then one triangle (a right triangle) can be formed. opp = height
   c. If $h < a < b$, then two triangles can be formed. opp between adj & height
   d. If $a \geq b$, then one triangle can be formed. opp ≥ adj



opp < height

(a) $a < h$

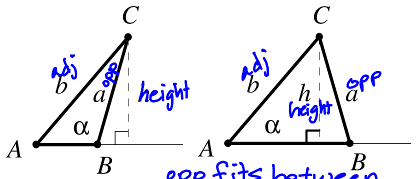
no triangle



opp = height

(b) $a = h$

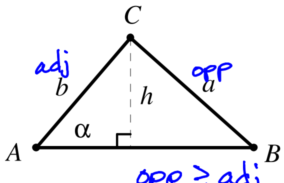
one triangle



opp fits between adjacent & height

(c) $h < a < b$

two triangles



opp ≥ adj

(d) $a \geq b$

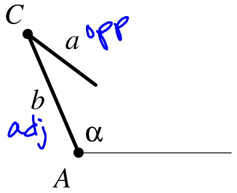
one triangle

If α is *obtuse*, there is either no triangle (if $a \leq b$) or one triangle (if $a > b$) possible.

opp ≤ adj

(a) $a \leq b$

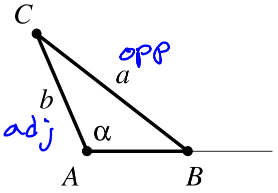
no triangle



opp > adj

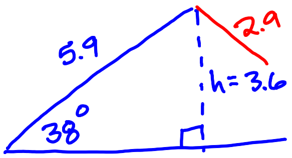
(b) $a > b$

one triangle



Examples:

a) $\beta = 38^\circ,\ b = \overset{\text{opp}}{2.9},\ c = \overset{\text{adj}}{5.9}$



$\sin 38^\circ = \frac{h}{5.9}$

$h = 5.9 \sin 38^\circ$

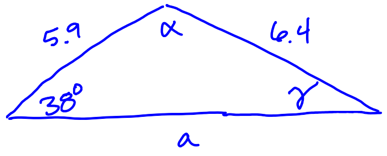
$h = 3.6$

$2.9 < 3.6$

opp < height $\Rightarrow$ $\boxed{\text{no triangle}}$

b) $\beta = 38^\circ,\ b = \overset{\text{opp}}{6.4},\ c = \overset{\text{adj}}{5.9}$

opp > adj $\Rightarrow$ 1 Δ



$\frac{\sin 38^\circ}{6.4} = \frac{\sin \gamma}{5.9}$

$\sin \gamma = \frac{5.9 \sin 38^\circ}{6.4} = .5676$

$\gamma = \sin^{-1}(.5676)$

$\boxed{\gamma = 34.6^\circ}$

$\alpha = 180^\circ - (38^\circ + 34.6^\circ)$

$\boxed{\alpha = 107.4^\circ}$

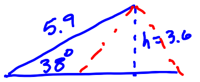
$\frac{\sin 38^\circ}{6.4} = \frac{\sin 107.4^\circ}{a}$

$a \sin 38^\circ = 6.4 \sin 107.4^\circ$

$a = \frac{6.4 \sin 107.4^\circ}{\sin 38^\circ}$

$\boxed{a = 9.9}$

c) $\beta = 38^\circ,\ b = \overset{\text{opp}}{4.7},\ c = \overset{\text{adj}}{5.9}$



h = 3.6 (see part a)

4.7 fits between 3.6 & 5.9

opp btwn adj & height

2 Δs

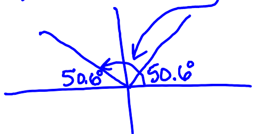
$\frac{\sin 38^\circ}{4.7} = \frac{\sin \gamma}{5.9}$

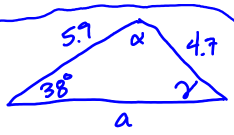
$\sin \gamma = \frac{5.9 \sin 38^\circ}{4.7}$

$\sin \gamma = .7729$

$\gamma = \sin^{-1}(.7729) = 50.6^\circ$

or $\gamma = 180^\circ - 50.6^\circ = 129.4^\circ$





$\boxed{\gamma = 50.6^\circ}$

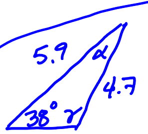
$\alpha = 180^\circ - (38^\circ + 50.6^\circ)$

$\boxed{\alpha = 91.4^\circ}$

$\frac{\sin 38^\circ}{4.7} = \frac{\sin 91.4^\circ}{a}$

$a \sin 38^\circ = 4.7 \sin 91.4^\circ$

$\boxed{a = 7.6}$



$\boxed{\gamma = 129.4^\circ}$

$\alpha = 180^\circ - (129.4^\circ + 38^\circ)$

$\boxed{\alpha = 12.6^\circ}$

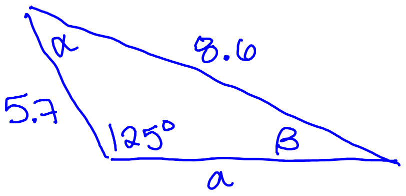
$\frac{\sin 38^\circ}{4.7} = \frac{\sin 126^\circ}{a}$

$a \sin 38^\circ = 4.7 \sin 12.6^\circ$

$\boxed{a = 1.7}$

d) $\gamma = 125^\circ$, $\overset{\text{adj}}{b = 5.7}$, $\overset{\text{opp}}{c = 8.6}$

opp > adj ⇒ one △



$$\frac{\sin 125^\circ}{8.6} = \frac{\sin\beta}{5.7}$$

$$\sin\beta = \frac{5.7\sin 125^\circ}{8.6}$$

$$\sin\beta \approx .5429$$

$$\boxed{\beta \approx 32.9^\circ}$$

$$\alpha = 180^\circ - (125^\circ + 32.9^\circ)$$

$$\boxed{\alpha \approx 22.1^\circ}$$

$$\frac{\sin 125^\circ}{8.6} = \frac{\sin 22.1^\circ}{a}$$

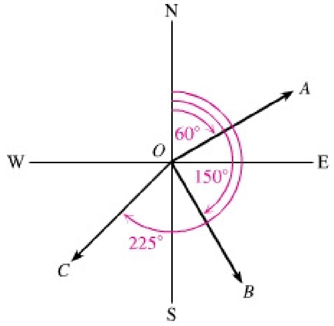
$$a\sin 125^\circ = 8.6\sin 22.1^\circ$$

$$\boxed{a \approx 4.0}$$

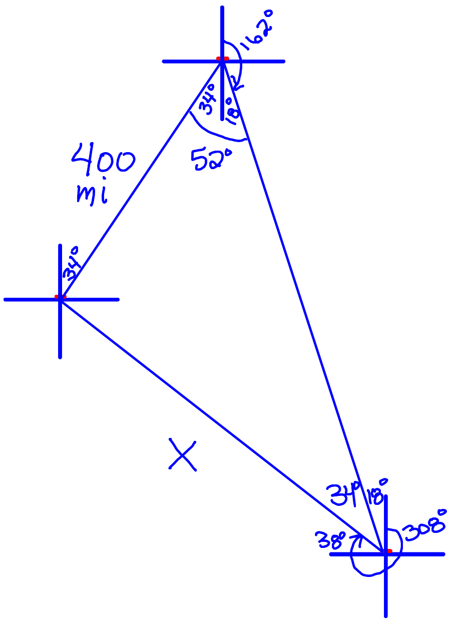
Bearing: The measure of an angle that describes the direction of a ray is called the bearing. Bearing is the clockwise angle from due north.

Another way to express bearing is to describe the acute angle that the ray makes with a ray pointing due north or south. For example:

N60°E is a bearing of 60° east of north
S30°E is a bearing of 30° east of south
S45°W is a bearing of 45° west of south



Example: During an important NATO exercise, an F-14 Tomcat left the carrier Nimitz on a course with a bearing of 34° and flew 400 miles. Then the F-14 flew for some distance on a course with a bearing of 162°. Finally, the plane flew back to its starting point on a course with a bearing of 308°. What distance did the plane fly on the final leg of the journey? Round to the nearest tenth of a mile.



$$\frac{\sin 34^\circ}{400} = \frac{\sin 52^\circ}{x}$$

$$x\sin 34^\circ = 400\sin 52^\circ$$

$$x = \frac{400\sin 52^\circ}{\sin 34^\circ}$$

$$\boxed{x \approx 563.7 \text{ mi}}$$