Definite Integrals and Antiderivatives

As it turns out, there are some really nice rules for working with definite integrals.
Remember when we partitioned the interval [a,b] we said $\Delta x = x_{k+1} - x_k$

Using the same function but integrating in the opposite order $\int_b^a f(x)\,dx$ has the effect of making Δx negative because x_k's decrease so $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$

What about $\int_a^a f(x)\,dx \quad dx = 0$

Properties of Definite Integrals

1. Order of Integration $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$

2. Zero $\int_a^a f(x)\,dx = 0$

3. Constant Multiple

$$\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx \qquad and \qquad \int_a^b -f(x)\,dx = -\int_a^b f(x)\,dx \quad when \quad k = -1$$

4. Sum and Difference $\int_a^b f(x) \pm g(x)\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

5. Additivity $\int_a^b f(x)dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$

6. Max-Min Inequality: If max *f* and min *f* are the maximum and minimum values of *f* on [a,b], then

$$\min f \cdot (b-a) \le \int_a^b f(x)\,dx \le \max f \cdot (b-a)$$

7. Domination:

$$f(x) \ge g(x) \quad on[a,b] \Rightarrow \int_a^b f(x)\,dx \ge \int_a^b g(x)\,dx$$

$$f(x) \ge 0 \quad on[a,b] \Rightarrow \int_a^b f(x)\,dx \ge 0 \qquad when \quad g = 0$$

Example 1 and Example 2

Now let's do #1

Average value of a Function

The average value of a function is exactly what it says. What is the function's average value? The problem with finding it using normal "averaging" methods is that there are infinitely many function values on any interval. Do something clever. Think of the values as defining an area, an integral…

$\int_a^b f(x)\,dx$ figures out the area.

What is the average height of the rectangles? You divide the area by b – a

Definition: Average (Mean) Value

If *f* is integrable on [a,b], its average (mean) value on [a,b] is $av(f) = \frac{1}{b-a}\int_a^b f(x)\,dx$

Example: Find the average value of $f(x) = x^2 + 2$ on [-1, 2] Use fnint (x^2+2, x, -1, 2)

$$\frac{1}{2-(-1)}\int_{-1}^{2} x^2 + 2 = \frac{1}{3}(9) = 3$$

Does *f(x)* actually have this value on this interval? Where? *f(x)* = 3 $x^2 + 2 = 3$ when x = ± 1
We are actually guaranteed an answer because of the mean value theorem for definite integrals.

Theorem 3

If *f* is continuous on [a,b], then at some point c in [a,b], $f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$

Let's read the two paragraphs under **Connecting Differential and Integral Calculus**

Do Exploration 2 (in groups)

If all went well in Exploration 2, you concluded that the derivative with respect to x of the integral of *f* from *a* to x is simply *f*….or $\frac{d}{dx}\int_a^x f(t)dt = f(x)$

This means that the integral is an antiderivative of *f*, a fact we can exploit in the following way.
If F is any antiderivative of *f* then $\int_a^x f(t)\ dt = F(x) + C$ for some constant C Setting x in this equation equal to *a* gives

$$\int_a^a f(t)\ dt = F(a) + C \qquad 0 = F(a) + C \qquad C = -F(a)$$

Putting it all together: $\int_a^x f(t)\ dt = F(x) - F(a)$
The antiderivative evaluated at x minus the antiderivative evaluated at *a*.

This is a big deal! We don't have to use Riemann sums, we can use antiderivatives!

$$\int_0^x \sin x\,dx$$

$$\int_0^3 x^2\ dx$$

$$\int_0^2 2x\,dx$$

$$\int_0^2 (4t-3)\,dt$$

$$\int_2^1 z^3 - 2z^2 + 4\ \ dz$$

$$\int_3^4 \frac{dx}{x}$$