Solving Trigonometric Equations

Basic steps for solving cos $x = a$:

1. **Find all the angles on the unit circle (on $[0, 2\pi]$) that satisfy the equation.** If a is not a unit circle value, use $\cos^{-1}(a)$ to find one of the angles, then figure out any other angles with the same cosine (draw the angle and figure out the other angle with the same x-coordinate on the unit circle).
2. **Add or subtract $2\pi k$ from each angle, where k is any integer.**

Basic steps for solving sin $x = a$:

1. **Find all the angles on the unit circle (on $[0, 2\pi]$) that satisfy the equation.** If a is not a unit circle value, use $\sin^{-1}(a)$ to find one of the angles (if you get a negative angle, add 2π), then figure out any other angles with the same sine (draw the angle and figure out the other angle with the same y-coordinate on the unit circle).
2. **Add or subtract $2\pi k$ from each angle, where k is any integer.**

Basic steps for solving tan $x = a$:

1. **Find one angle on the unit circle that satisfies the equation.** If a is not a unit circle value, use $\tan^{-1}(a)$ to find an angle. You don't need to find another angle, because unlike sine and cosine, tangent repeats at regular intervals.
2. **Add or subtract multiples of π from each angle.** (Tangent repeats every π instead of every 2π like sine and cosine).

Examples: Find all real numbers (that means radians) that satisfy each equation.

a) $\sin x = 1$

b) $\cos x = 0$

c) $\cos x = -1/2$

d) $\sin x = \sqrt{2}/2$

e) $\tan x = -\sqrt{3}$

f) $\tan x = 1$

g) $\sin x = -.4375$

h) $\cos x = .8913$

Examples: Find all angles in $[0°,\ 360°]$ that satisfy each equation

a) $\cos x = \sqrt{3}/2$

b) $\tan x = -3.5$

Sometimes, you have to do a bit of algebra before you can use the techniques above.

a) Solve $2\cos\alpha - 1 = 0$ for $0 \le \alpha \le 2\pi$.

b) Solve $3\sin(\beta) + 6 = 5\sin(\beta) + 7$ for $0^\circ \le \beta \le 360^\circ$.

Solving Multiple-Angle Equations

Often, equations involve expressions like $\sin 2x$, $\cos 3\alpha$, or $\tan(x/2)$, all of which involve multiples of the variable rather than a single variable.

1. Solve for the multiple variable just as we would solve for a single variable. (eg. Solve for $2x$)
2. Multiply or divide to get the single variable in the last step. (eg. Divide by 2 to solve for x)

Example: Find all solutions in degrees to $\sin 2\alpha = \sqrt{3}/2$.

Example: Find all solutions to $\tan(4x) = 1$ in the interval $(0, \pi)$.

Example: Find all real number solutions to $\cos(x/2) = -1/2$.

Example: Find all solutions to $\sec(3x) = 2\sqrt{3}/3$ in the interval $(0°, 360°)$.

The Path of a Projectile
The distance d (in feet) traveled by a projectile fired from the ground with an angle of elevation θ is related to the initial velocity v_0 (in ft/sec) by the equation $v_0^2 \sin 2\theta = 32d$. If the projectile is fired from the origin into the first quadrant, then the x- and y-coordinates (in feet) of the projectile at time t (in seconds) are given by $x = v_0 t \cos\theta$ and $y = -16t^2 + v_0 t \sin\theta$.

Example: A catapult is placed 100 feet from the castle wall, which is 35 feet high. A soldier wants a burning bale of hay to clear the top of the wall and land 50 feet inside the castle wall. If the initial velocity of the bale is 70 ft/sec, then at what angle should the bale of hay be launched so that it will travel 150 feet and pass over the castle wall?