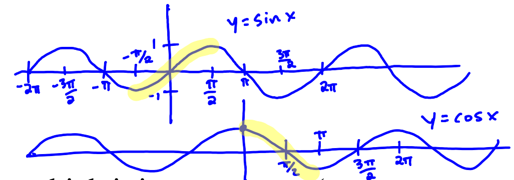
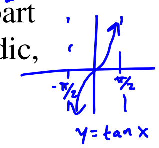
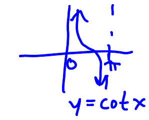
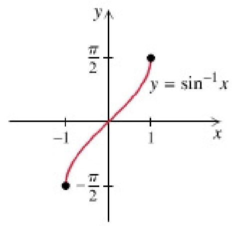


The Inverse Trigonometric Functions

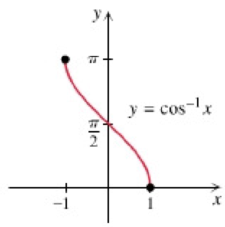




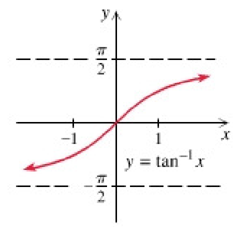
We can only talk about the inverse of a function on a part of its domain on which it is one-to-one (a part of the domain on which it passes the horizontal line test). Since the trigonometric functions are periodic, we must pick a part of their domain on which they are one-to-one in order to define the inverse functions. The domain of the inverse functions is the values of the sine, cosine, tangent, etc. and the range is the measures of *one set of possible angles* that have those specific sines, cosines, tangents, etc.



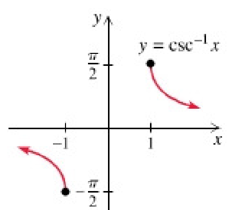
Domain $[-1, 1]$
Range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



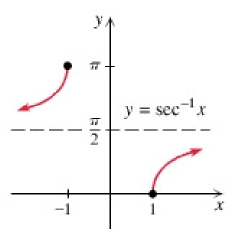
Domain $[-1, 1]$
Range $[0, \pi]$



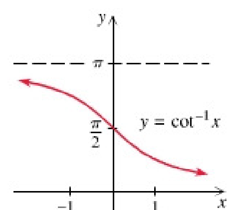
Domain $(-\infty, \infty)$
Range $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



Domain $(-\infty, -1] \cup [1, \infty)$
Range $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$

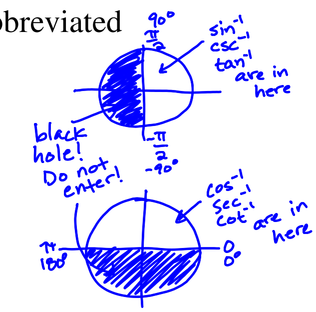


Domain $(-\infty, -1] \cup [1, \infty)$
Range $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$



Domain $(-\infty, \infty)$
Range $(0, \pi)$

The **inverse sine function** is sometimes called the **arc sine**, and is abbreviated $\arcsin(x)$ or $\sin^{-1}(x)$. Similarly, the other inverse functions are often called the **arc cosine** and **arc tangent**, abbreviated $\arccos(x)$ or $\cos^{-1}(x)$ and $\arctan(x)$ or $\tan^{-1} x$.



- $\sin^{-1}(x)$ or $\arcsin(x)$ is the angle in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ whose sine is *x*. $[-90°, 90°]$
- $\cos^{-1}(x)$ or $\arccos(x)$ is the angle in $[0, \pi]$ whose cosine is *x*. $[0°, 180°]$
- $\tan^{-1}(x)$ or $\arctan(x)$ is the angle in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ whose tangent is *x*. $(-90°, 90°)$

Example: Find the exact value of each expression without using a table or calculator.

a) $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ b) $\arcsin\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$ c) $\tan^{-1}(1) = \frac{\pi}{4}$ d) $\arccos\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

Do not write $\frac{7\pi}{4}$!

$\frac{y}{x}$ > x & y same

Example: Find the measure of angle α.

a) $\sin\alpha = 0.56,\ -90° \le \alpha \le 90°$

b) $\tan\alpha = -3,\ -\pi/2 < \alpha < \pi/2$

c) $\cos\alpha = 0.23,\ 0° \le \alpha \le 180°$

d) $\cos\alpha = -0.82,\ 0 \le \alpha \le \pi$

Inverses of the Reciprocal Trigonometric Functions

- $\csc^{-1}(x)$ or $\operatorname{arccsc}(x)$ is the angle in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ whose cosecant is *x*.
- $\sec^{-1}(x)$ or $\operatorname{arcsec}(x)$ is the angle in $[0, \pi]$ whose secant is *x*.
- $\cot^{-1}(x)$ or $\operatorname{arccot}(x)$ is the angle in $(0, \pi)$ whose cotangent is *x*.

Identities

- $\csc^{-1}(x) = \sin^{-1}\left(\frac{1}{x}\right)$ If $\csc\theta = x$, $\sin\theta = \frac{1}{x}$
- $\sec^{-1}(x) = \cos^{-1}\left(\frac{1}{x}\right)$
- $\cot^{-1}(x) = \begin{cases} \tan^{-1}(1/x) & \text{for } x > 0 \\ \tan^{-1}(1/x) + \pi & \text{for } x < 0 \\ \pi/2 & \text{for } x = 0 \end{cases}$
- $\cot^{-1}(x) = \frac{\pi}{2} - \tan^{-1}(x)$

$\frac{2}{\sqrt{2}} = \sqrt{2}$ ← comes from flipping $\sqrt{2}/2$ upside down

$\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$ ← comes from flipping $\frac{\sqrt{3}}{2}$ upside down

$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ ← comes from $\frac{1/2}{\sqrt{3}/2}$

$\sqrt{3}$ ← come from $\frac{\sqrt{3}/2}{1/2}$

Example: Find the exact value of each expression without using a table or calculator.

a) $\operatorname{arcsec}(-2)$

$\sec\theta = -2$

$\cos\theta = -1/2$

$[0°, 180°]$

$\boxed{120°}$

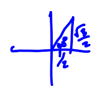
b) $\csc^{-1}\left(\sqrt{2}\right)$

$\sin\theta = \frac{\sqrt{2}}{2}$

$\boxed{45°}$

c) $\operatorname{arccot}\left(\frac{\sqrt{3}}{3}\right)$

$\cot\theta = \frac{1}{\sqrt{3}}$



$\frac{x}{y} = \frac{1/2}{\sqrt{3}/2}$

$\boxed{60°}$

Example: Find the approximate value of each expression rounded to 4 decimal places.

a) $\operatorname{arccsc}(-1.4713)$

$\csc\theta = -1.4713$

$\Rightarrow \sin\theta = -\frac{1}{1.4713}$

$\arcsin\left(-\frac{1}{1.4713}\right)$

$\approx \boxed{-.7473 \text{ rad}}$

b) $\cot^{-1}(-2.5)$ ← must be between 0 & π

$\cot\theta = -2.5$

$\tan\theta = -\frac{1}{2.5}$

$\tan^{-1}\left(-\frac{1}{2.5}\right) \approx -0.3805$ ← not between 0 & π

tan & cot repeat every π radians → just add π!

$-.3805 + \pi \approx \boxed{2.7611}$

c) $\sec^{-1}(4.328)$

$\cos^{-1}\left(\frac{1}{4.328}\right)$

$\approx \boxed{1.3376 \text{ rad}}$

Example: Find the exact value of each composition.

a) $\sin\left(\cot^{-1}(-1)\right)$

$= \sin\left(\frac{3\pi}{4}\right)$

$= \boxed{\frac{\sqrt{2}}{2}}$

b) $\sin^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right)$

$= \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

$= \boxed{-\frac{\pi}{4}}$

c) $\arcsin\left(\cos\left(\frac{\pi}{6}\right)\right)$

$= \arcsin\left(\frac{\sqrt{3}}{2}\right)$

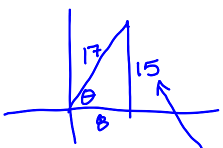
$= \boxed{\frac{\pi}{3}}$

Examples: Find the exact value of each composition.

a) $\sin\left(\cos^{-1}\left(\frac{8}{17}\right)\right)$

Asking question: What is $\sin\theta$?

Giving info: $\cos\theta = 8/17$, $0° \le \theta \le 180°$



$y = \sqrt{17^2 - 8^2} = 15$

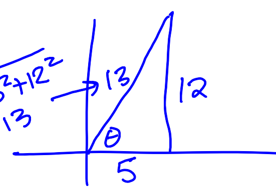
$\sin\theta = \boxed{\frac{15}{17}}$

b) $\sec\left(\operatorname{arccot}\left(\frac{5}{12}\right)\right)$

What is $\sec\theta$?

$\cot\theta = \frac{5}{12} \leftarrow \frac{x}{y}$

$0° \le \theta \le 180°$

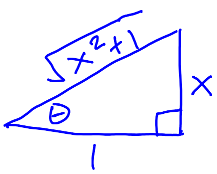


$r = \sqrt{5^2 + 12^2} = 13$

$\sec\theta = \boxed{\frac{13}{5}}$ ← $\frac{r}{x}$

Example: Find an equivalent algebraic expression for $\sin(\arctan(x))$

What is $\sin\theta$? $\tan\theta = \frac{x}{1}$ $\frac{\text{opp}}{\text{adj}}$



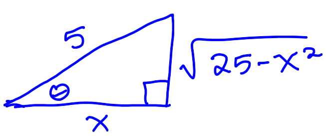
$x^2 + 1^2 = \text{hyp}^2$

$\text{hyp} = \sqrt{x^2+1}$

$\sin\theta = \frac{x}{\sqrt{x^2+1}}$

Example: Find an equivalent algebraic expression for $\cot\left(\arccos\left(\frac{x}{5}\right)\right)$

What is $\cot\theta$? $\cos\theta = \frac{x}{5}$ $\frac{\text{adj}}{\text{hyp}}$



$\text{leg} = \sqrt{\text{hyp}^2 - \text{leg}^2}$

$= \sqrt{5^2 - x^2}$

$= \sqrt{25 - x^2}$

$\cot\theta = \boxed{\frac{x}{\sqrt{25-x^2}}}$