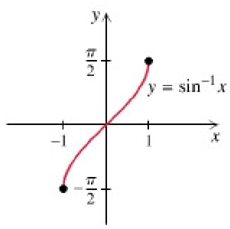
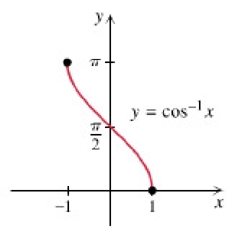
The Inverse Trigonometric Functions

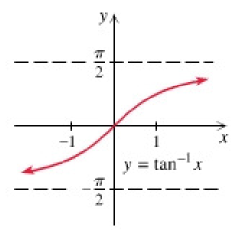
We can only talk about the inverse of a function on a part of its domain on which it is one-to-one (a part of the domain on which it passes the horizontal line test). Since the trigonometric functions are periodic, we must pick a part of their domain on which they are one-to-one in order to define the inverse functions. The domain of the inverse functions is the values of the sine, cosine, tangent, etc. and the range is the measures of *one set of possible angles* that have those specific sines, cosines, tangents, etc.



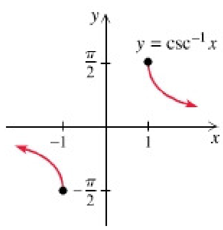
Domain $[-1, 1]$
Range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



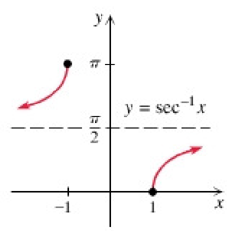
Domain $[-1, 1]$
Range $[0, \pi]$



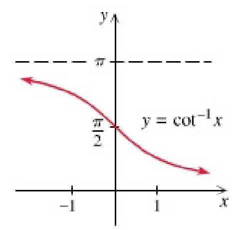
Domain $(-\infty, \infty)$
Range $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



Domain $(-\infty, -1] \cup [1, \infty)$
Range $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$



Domain $(-\infty, -1] \cup [1, \infty)$
Range $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$



Domain $(-\infty, \infty)$
Range $(0, \pi)$

The **inverse sine function** is sometimes called the **arc sine**, and is abbreviated $\arcsin(x)$ or $\sin^{-1}(x)$. Similarly, the other inverse functions are often called the **arc cosine** and **arc tangent**, abbreviated $\arccos(x)$ or $\cos^{-1}(x)$ and $\arctan(x)$ or $\tan^{-1} x$.

- $\sin^{-1}(x)$ or $\arcsin(x)$ is the angle in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ whose sine is *x*.
- $\cos^{-1}(x)$ or $\arccos(x)$ is the angle in $[0, \pi]$ whose cosine is *x*.
- $\tan^{-1}(x)$ or $\arctan(x)$ is the angle in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ whose tangent is *x*.

Example: Find the exact value of each expression without using a table or calculator.

a) $\sin^{-1}\left(\frac{1}{2}\right)$ b) $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$ c) $\tan^{-1}(1)$ d) $\arccos\left(-\frac{1}{2}\right)$

Example: Find the measure of angle α.

a) $\sin\alpha = 0.56,\ -90° \le \alpha \le 90°$

b) $\tan\alpha = -3,\ -\pi/2 < \alpha < \pi/2$

c) $\cos\alpha = 0.23,\ 0° \le \alpha \le 180°$

d) $\cos\alpha = -0.82,\ 0 \le \alpha \le \pi$

Inverses of the Reciprocal Trigonometric Functions

- $\csc^{-1}(x)$ or $\operatorname{arccsc}(x)$ is the angle in $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ whose cosecant is *x*.
- $\sec^{-1}(x)$ or $\operatorname{arcsec}(x)$ is the angle in $[0,\pi]$ whose secant is *x*.
- $\cot^{-1}(x)$ or $\operatorname{arccot}(x)$ is the angle in $(0,\pi)$ whose cotangent is *x*.

Identities

- $\csc^{-1}(x)=\sin^{-1}\left(\frac{1}{x}\right)$
- $\sec^{-1}(x)=\cos^{-1}\left(\frac{1}{x}\right)$
- $\cot^{-1}(x)=\begin{cases}\tan^{-1}(1/x) & \text{for } x>0\\ \tan^{-1}(1/x)+\pi & \text{for } x<0\\ \pi/2 & \text{for } x=0\end{cases}$
- $\cot^{-1}(x)=\frac{\pi}{2}-\tan^{-1}(x)$

Example: Find the exact value of each expression without using a table or calculator.

a) $\operatorname{arcsec}(-2)$ b) $\csc^{-1}\left(\sqrt{2}\right)$ c) $\operatorname{arccot}\left(\frac{\sqrt{3}}{3}\right)$

Example: Find the approximate value of each expression rounded to 4 decimal places.

a) $\operatorname{arccsc}(-1.4713)$ b) $\cot^{-1}(-2.5)$ c) $\sec^{-1}(4.328)$

Example: Find the exact value of each composition.

a) $\sin\left(\cot^{-1}(-1)\right)$ b) $\sin^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right)$ c) $\arcsin\left(\cos\left(\frac{\pi}{6}\right)\right)$

Examples: Find the exact value of each composition.

a) $\sin\left(\cos^{-1}\left(\frac{8}{17}\right)\right)$

b) $\sec\left(\operatorname{arccot}\left(\frac{5}{12}\right)\right)$

Example: Find an equivalent algebraic expression for $\sin(\arctan(x))$

Example: Find an equivalent algebraic expression for $\cot\left(\arccos\left(\frac{x}{5}\right)\right)$