

5.4

Precalculus Identities – Double angle and Half-angle

Name_____ Date_____ Per_____

Find the exact value of each expression using double-angle identities. Show work (do not use the unit circle).

1. $\sin(90^\circ)$

2. $\tan(60^\circ)$

3. $\cos\left(\frac{4\pi}{3}\right)$

4. $\sin\left(\frac{2\pi}{3}\right)$

Use the appropriate identities to simplify each expression to a single trigonometric function. If the trigonometric function is on the unit circle, write the ratio of the trigonometric function. Do not use a calculator. You can use the unit circle.

5. $2\sin(13^\circ)\cos(13^\circ)$

6. $2\cos^2(22.5^\circ) - 1$

7. $\frac{2\tan(15^\circ)}{1 - \tan^2(15^\circ)}$

8. $\cos^2\left(\frac{\pi}{9}\right) - \sin^2\left(\frac{\pi}{9}\right)$

Find the exact value of each expression using the half-angle identities. Make sure to use the correct sign.

9. $\cos(22.5^\circ)$

10. $\sin(-3\pi/8)$

11. $\tan(75^\circ)$

12. $\sin(105^\circ)$

13. $\cos(7\pi/12)$

14. $\tan(-5\pi/12)$

Use identities to simplify each expression.

15. $2\sin 105^\circ \cos 105^\circ$

16. $2\cos^2(22.5^\circ) - 1$

17. $\cos^2(\pi/9) - \sin^2(\pi/9)$

Find the values of $\sin \alpha$, $\cos \alpha$, $\tan \alpha$, $\csc \alpha$, $\sec \alpha$, and $\cot \alpha$.

18. $\cos(2\alpha) = 3/5$ and $0^\circ < 2\alpha < 90^\circ$

19. $\sin(2\alpha) = -8/17$ and $180^\circ < 2\alpha < 270^\circ$

20. $\sin(\alpha/2) = 4/5$ and $\alpha/2$ is in Quadrant II

21. $\cos(\alpha/2) = -1/4$ and $\pi/2 < \alpha/2 < 3\pi/4$

Solve each problem.

22. Find the exact value of $\sin(2\alpha)$ given that $\sin(\alpha) = 7/25$ and α is in Quadrant I.

23. Find the exact value of $\cos(2\alpha)$ given that $\cos(\alpha) = -8/17$ and α is in Quadrant II.

24. Find the exact value of $\tan(2\alpha)$ given that $\sin(\alpha) = -4/5$ and α is in Quadrant III.

Prove that each equation is an identity.

25. $\cos^4 \theta - \sin^4 \theta = \cos(2\theta)$

26. $\frac{\sin(4x)}{4} = \cos^3 x \sin x - \sin^3 x \cos x$

$$27. \frac{\cos(2x) + \cos(2y)}{\sin x + \cos y} = 2\cos y - 2\sin x$$

$$28. (\sin \alpha - \cos \alpha)^2 = 1 - \sin(2\alpha)$$

$$29. \frac{\cos(2x)}{\sin^2 x} = \csc^2 x - 2$$