Connecting f′ and f″ with the graph of f

We already know that when:
f′ (c) = 0 c is a possible max or min
f′ (c) > 0 at c, *f* is increasing
f′ (c) < 0 at c, *f* is decreasing
Look at Figure 5.18

Check out: First Derivative Test for Local Extrema
If f′ changes sign from positive to negative at c then f has a local maximum value at c.
If f′ changes sign from negative to positive at c then f has a local minimum value at c.
If f′ does not change sign at c, then f has no local extreme value at c.
At a left endpoint a: If f′ < 0 for x>a, then f has a local maximum value at a. If f ′ > 0 for x > a, then f has a local minimum value at a.
At a right endpoint b: If f′ < 0 for x<b, then f has a local minimum value at a. If f ′ > 0 for x < b, then f has a local maximum value at a.

Look at figure 5.21
There is a nice definition of concavity.
Concave up when y′ is increasing
Concave down when y′ is decreasing

We are really talking about how y′ is changing.
Concave up when y″ > 0
Concave down when y″ < 0
Changes concavity when y″ = 0

Now look at page 212 figure 5.22 and 5.23

Points of Inflection
A **point of inflection** is a point where the graph of a function has a tangent line and where the concavity changes.

Critical Points
f′ (c) Does not exist (in the function this is at a corner, cusp, jump, not if the point is a vertical tangent)
f′ (c) = 0 possible max or min
Remember that max or min can occur at an endpoint of the interval.
f″ (c) = 0 point of inflection. Changes from concave up to down or down to up; but it must have a tangent line.

It is good to note that when a function is increasing (concave up) the tangent line is below the curve.
When it is concave down, the tangent line is above the curve.

Example: $y = x^3 - 12x - 5$ $\qquad y' = 3x^2 - 12$ $\qquad y'' = 6x$

Test the critical points:
f′ (2) = 0 and f″(2)>0 then f has a local minimum at x = 2
f′ (-2) = 0 and f″(-2)<0 then f has a local maximum at x = -2
f′ (0) < 0 and f″(0)=0 then point of inflection

Example: $y = x^3 - 12x - 5$ $y' = 3x^2 - 12$ $y'' = 6x$

Find intervals where function is increasing and decreasing

Intervals	x<-2	X=-2	-2< x<0	0<x<2	X=2	x>2
f′	Positive	0	Negative	negative	0	positive
f″	Negative	negative	Negative	positive	positive	positive
Function	Increasing	positive	decreasing	decreasing	negative	increasing

Second Derivative Test for Local Extrema

If f′ (c) = 0 and f″(c)<0 then f has a local maximum at x = c

If f′ (c) = 0 and f″(c)>0 then f has a local minimum at x = c

Example 8

Exploration 1 page 217

Learn about functions from Derivatives.

Exploration 2 page 218

Examples:

Use analytic methods to find the intervals on which the function is increasing, decreasing, concave up and concave down. Also find local extreme values, and inflection points.

3. $y = 2x^4 - 4x^2 + 1$

51. f is continuous on [0,3] and satisfies the following. Find the absolute extrema of f and where they occur; find any points of inflection, sketch a possible graph of f.

x	0	1	2	3
f	0	2	0	-2
f′	3	0	does not exist	-3
f″	0	-1	does not exist	0

x	0<x<1	1<x<2	2<x<3
f	+	+	-
f′	+	-	-
f″	-	-	-