Verifying Identities

a.k.a. Proofs

A General Strategy for Verifying Identities

1. Work on the more complicated side first.
2. Rewrite the side you are working with in terms of sines and cosines only.
3. Write a single rational expression as a sum of two rational expressions.
4. Combine a sum of two rational expressions into a single rational expression.
5. If both sides simplify to a third expression, then the equation is an identity.
6. Multiply the numerator and denominator of one rational expression by either the numerator or denominator of the other.

Do not skip steps or take shortcuts! You must give a logical sequence of connected steps proving that one side of the identity is equal to the other. Your work is your answer. If you combine too much in one step, you will lose points. If you do steps in your head without showing work, you will lose points. Also, don't connect things that aren't equal with equal signs!

Examples:

Verify that $1+\sec x \sin x \tan x = \sec^2 x$ is an identity.

$$1 + \sec x \sin x \tan x$$
$$= 1 + \left(\frac{1}{\cos x}\right)\left(\frac{\sin x}{1}\right)\left(\frac{\sin x}{\cos x}\right)$$
$$= 1 + \frac{\sin^2 x}{\cos^2 x}$$
$$= 1 + \tan^2 x$$
$$= \sec^2 x$$

Q.E.D. → quod erat demonstratum

"And thus it is proven"

Prove that $\dfrac{\csc x - \sin x}{\sin x} = \cot^2 x$ is an identity.

$$\frac{\csc x - \sin x}{\sin x}$$
$$= \frac{\frac{1}{\sin x} - \sin x}{\sin x}$$
$$= \left(\frac{\frac{1}{\sin x} - \sin x}{\sin x}\right)\left(\frac{\sin x}{\sin x}\right)$$
$$= \frac{1 - \sin^2 x}{\sin^2 x}$$
$$= \frac{\cos^2 x}{\sin^2 x}$$
$$= \cot^2 x$$

Q.E.D.

Prove that $-2\cot^2 x = \dfrac{1}{1-\sec x} + \dfrac{1}{1+\sec x}$ is an identity.

$\tan^2 x + 1 = \sec^2 x$

$1 - \sec^2 x = -\tan^2 x$

$$= \left(\frac{1+\sec x}{1+\sec x}\right)\left(\frac{1}{1-\sec x}\right) + \left(\frac{1}{1+\sec x}\right)\left(\frac{1-\sec x}{1-\sec x}\right)$$
$$= \frac{1+\sec x + 1 - \sec x}{(1+\sec x)(1-\sec x)}$$
$$= \frac{2}{1-\sec^2 x}$$
$$= \frac{2}{-\tan^2 x}$$
$$= -2\cot^2 x$$

Verify that $\dfrac{1-\cos^2(-t)}{\sin(-t)} = \tan(-t)\cos(-t)$ is an identity.

$$= \frac{1-\cos^2 t}{-\sin t}$$

$$= \frac{\sin^2 t}{-\sin t}$$

$$= -\sin t$$

$$= (-\tan t)(\cos t)$$

$$= \left(-\frac{\sin t}{\cancel{\cos t}}\right)(\cancel{\cos t})$$

$$= -\sin t$$

Q.E.D.

Show that $\dfrac{1-\sin^2 t}{1-\csc(-t)} = \dfrac{1+\sin(-t)}{\csc t}$ is an identity.

$$= \frac{1-\sin^2 t}{1-(-\csc t)}$$

$$= \frac{1-\sin^2 t}{1+\frac{1}{\sin t}}$$

$$= \left(\frac{1-\sin^2 t}{1+\frac{1}{\sin t}}\right)\left(\frac{\sin t}{\sin t}\right)$$

$$= \frac{(1-\sin^2 t)(\sin t)}{\sin t + 1}$$

$$= \frac{\cancel{(1+\sin t)}(1-\sin t)(\sin t)}{\cancel{\sin t + 1}} = (1-\sin t)(\sin t)$$

$$= \frac{1-\sin t}{\csc t}$$

$$= \frac{1-\sin t}{\frac{1}{\sin t}}$$

$$= (1-\sin t)(\sin t)$$

Q.E.D.

Prove that $\dfrac{\cos\alpha}{1-\sin\alpha} = \dfrac{1+\sin\alpha}{\cos\alpha}$ is an identity.

$$= \left(\frac{\cos\alpha}{1-\sin\alpha}\right)\left(\frac{1+\sin\alpha}{1+\sin\alpha}\right)$$

$$= \frac{\cos\alpha(1+\sin\alpha)}{1-\sin^2\alpha}$$

$$= \frac{\cos\alpha(1+\sin\alpha)}{\cos^2\alpha}$$

$$= \frac{1+\sin\alpha}{\cos\alpha}$$

Q.E.D.