Precalculus
5.2 Homework
Odd Answers

★ **Note: These are not the only possible answers! There may even be more efficient proofs. I have a talent for making proofs more complicated than they need to be.**

1. $\sin x \cot x$

$$= \sin x \left(\frac{\cos x}{\sin x} \right)$$

$$= \cos x$$

3. $\dfrac{\cos x \sin^2 x + \cos^3 x}{\sin x}$

$$= \frac{\cos x \left(\sin^2 x + \cos^2 x \right)}{\sin x}$$

$$= \frac{(\cos x)(1)}{\sin x}$$

$$= \frac{\cos x}{\sin x} = \cot x$$

5. $\csc x \sin^2 x - \tan(-x)\cos(-x)$

$$= \csc x \sin^2 x - (-\tan x)\cos x$$

$$= \left(\frac{1}{\sin x} \right) \sin^2 x + \left(\frac{\sin x}{\cos x} \right) \cos x$$

$$= \sin x + \sin x$$

$$= 2\sin x$$

7. $(1 + \cot \alpha)^2 - 2\cot \alpha$

$$= 1 + 2\cot \alpha + \cot^2 \alpha - 2\cot \alpha$$
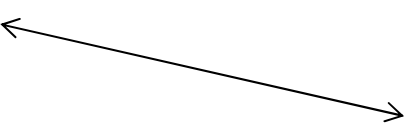
$$= 1 + \cot^2 \alpha$$

$$= \csc^2 \alpha$$

$$\dfrac{1}{(1 - \cos \alpha)(1 + \cos \alpha)}$$

$$= \frac{1}{1 - \cos^2 \alpha}$$

$$= \frac{1}{\sin^2 \alpha}$$

$$= \csc^2 \alpha$$

9.

$\tan x + \cot x$

$= \dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}$

$= \left(\dfrac{\sin x}{\sin x}\right)\left(\dfrac{\sin x}{\cos x}\right) + \left(\dfrac{\cos x}{\sin x}\right)\left(\dfrac{\cos x}{\cos x}\right)$

$= \dfrac{\sin^2 x + \cos^2 x}{\sin x \cos x}$

$= \dfrac{1}{\sin x \cos x}$

$= \dfrac{1}{\sin x} \cdot \dfrac{1}{\cos x}$

$= \csc x \sec x$

$= \sec x \csc x$

11.

$\dfrac{\csc x}{\csc x - \sin x}$

$= \dfrac{\frac{1}{\sin x}}{\frac{1}{\sin x} - \sin x}$

$= \left(\dfrac{\frac{1}{\sin x}}{\frac{1}{\sin x} - \sin x}\right)\left(\dfrac{\sin x}{\sin x}\right)$

$= \dfrac{1}{1 - \sin^2 x}$

$= \dfrac{1}{\cos^2 x}$

$= \sec^2 x$

13.

$\dfrac{\csc y + 1}{\csc y - 1}$

$= \dfrac{\frac{1}{\sin y} + 1}{\frac{1}{\sin y} - 1}$

$= \left(\dfrac{\frac{1}{\sin y} + 1}{\frac{1}{\sin y} - 1}\right)\left(\dfrac{\sin y}{\sin y}\right)$

$= \dfrac{1 + \sin y}{1 - \sin y}$