Extreme Values of Functions

Calculus is about rates of change and accumulation. We have studied rates of change (derivatives). That branch of mathematics is called differential calculus. Since we are now good at differentiating functions, let's learn to apply derivatives in useful ways.

In the past, one of the main uses of the derivative was to find information about a graph. This is generally called finding critical values of a graph. But we will do much more than that with derivatives.

If we talk about extreme temperatures, what are we talking about?
In calculus we are concerned with when a function reaches an absolute max or absolute min. (absolute = global)

Graphically, what do max and min have in common?

If the function has a local maximum value or local minimum value at an interior point of its domain and if $f'(c)$ exists, then $f'(c) = 0$

Look at Figure 5.2.

Theorem 1: The Extreme Value Theorem
If *f* is continuous on a closed interval [a, b], then f has both a maximum value and a minimum value on the interval.

A critical point(s) of a function occurs when:
f' (c) **does not exist**
f' (c) **= 0 possible point of max or min occurs**
Extreme values occur only at critical points and endpoints.
Never forget that a max or min can occur at an end point.

Definition: A point in the interior of the domain of a function *f* at which *f'* (c) = 0 is called a stationary point.

#17 $f(x) = x^{\frac{2}{5}} \quad -3 \le x < 1$

Example: extreme values: $f(x) = \dfrac{1}{\sqrt{4 - x^2}}$

Find them and say max or min. How is the slope changing on either side?

Look at page 210 Theorem 4: The first derivative test for local extrema

Examples:

$y = x^2 + 4x$

$y = x^3$

$y = x^{\frac{1}{3}}$

$f(x) = x^3 - 12x - 5$

$f(x) = (x^2 - 3)e^x$

This will be tied together by section 5.3

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Do #26 without a calculator:

#26 $y = \dfrac{1}{\sqrt[3]{1 - x^2}}$