Basic Trigonometric Identities

An ***equation*** is any mathematical statement involving an equal sign. There are three types of equations:

- ***Contradictions*** are equations that are never true, like $0=1$, or $x+5=x-7$.
- ***Conditional equations*** are equations that are sometimes true - true only for certain values of the variable(s) - like $x+5=7$, or $\sin\theta=\sqrt{3}/2$.
- ***Identities*** are equations that are true for all possible values of the variables, like $x+y=y+x$, or $A^2-B^2=(A+B)(A-B)$ or $\csc\theta=1/\sin\theta$.

Many trigonometric identities can be derived quickly from the *x*, *y*, *r* definitions of the trigonometric functions.

Reciprocal Identities:

$$\sin\theta=\frac{1}{\csc\theta}\qquad \cos\theta=\frac{1}{\sec\theta}\qquad \tan\theta=\frac{1}{\cot\theta}$$

$$\csc\theta=\frac{1}{\sin\theta}\qquad \sec\theta=\frac{1}{\cos\theta}\qquad \cot\theta=\frac{1}{\tan\theta}$$

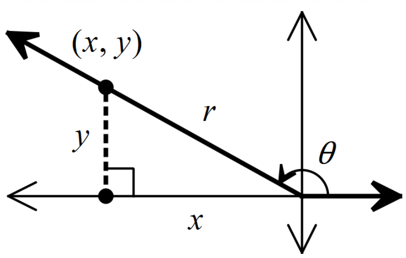
Tangent and Cotangent Quotient Identities:

$$\tan\theta=\frac{\sin\theta}{\cos\theta}\qquad \cot\theta=\frac{\cos\theta}{\sin\theta}$$

The Fundamental Identity

Remember that by definition, $\sin\theta=y/r$, $\cos\theta=x/r$, and by the Pythagorean Theorem, $x^2+y^2=r^2$.

$$\sin^2\theta+\cos^2\theta=\left(\frac{\quad}{\quad}\right)^2+\left(\frac{\quad}{\quad}\right)^2$$



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The Fundamental (Pythagorean) Identity: $\sin^2\theta+\cos^2\theta=\underline{\qquad}$

Pythagorean Identities: We can use the fundamental identity to derive two more identities. Together, these three identities are called Pythagorean Identities because they are derived from the Pythagorean Theorem:

$$\sin^2\theta+\cos^2\theta=1$$

$$\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta}=\frac{1}{\sin^2\theta}$$

$$\frac{\qquad}{\qquad}+\frac{\qquad}{\qquad}=\frac{\qquad}{\qquad}$$

$$\underline{\qquad}+\underline{\qquad}=\underline{\qquad}$$

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$$\underline{\qquad}+\underline{\qquad}=\underline{\qquad}$$

Simplifying Expressions

We can use the identities above to simplify trigonometric expressions. One of the most common strategies is to start by rewriting the expression in terms of sines and/or cosines, then simplify from there.

Examples: Simplify the following.

a) $\dfrac{\tan x}{\sec x}$ b) $\sin\alpha + \cot\alpha\cos\alpha$ c) $\dfrac{\tan\theta\csc\theta}{\sec\theta}$

Using Identities to Find Function Values

We know how to draw a triangle to find missing function values, but we can also find missing function values using identities.

Example: If $\tan\alpha = -2/3$ and α is in quadrant IV, use identities to find the values of the remaining five trigonometric functions.

Multiplying and Factoring Polynomials Involving Trigonometric Functions

We must often multiply or factor expressions involving trigonometric functions when we simplify or verify identities or solve trigonometric equations.

Examples:

a) Multiply $(1+\tan x)(1-\tan x)$

b) Multiply $(2\sin x + 1)^2$

c) Factor $2\sin x\cos x + \cos x$

d) Factor $\sec^2 x - \tan^2 x$

e) Factor $\sin^2 x + \sin x - 2$

f) Factor $3\cos^2 x - 7\cos x - 6$

Odd and Even Identities

Odd functions have graphs that are symmetric with respect to the ____________________.
Even functions have graphs that are symmetric with respect to the ____________________.

In odd functions, $f(-x) =$ _______. In even functions, $f(-x) =$ _______.

Sketch the graphs of the six parent functions below, and decide which are odd and which are even.

$f(x) = \sin x$ $\quad f(x) = \cos x$ $\quad f(x) = \tan x$ $\quad f(x) = \csc x$ $\quad f(x) = \sec x$ $\quad f(x) = \cot x$

Fill in the blanks to complete the odd and even identities:

$\cos(-x) =$ ____________ $\quad \sin(-x) =$ ____________ $\quad \tan(-x) =$ ____________

$\sec(-x) =$ ____________ $\quad \csc(-x) =$ ____________ $\quad \cot(-x) =$ ____________

Examples: Simplify the following.

a) $\csc(-x)\tan(-x)$

b) $\dfrac{1}{1+\cos(-x)} + \dfrac{1}{1-\cos x}$

c) $\tan^2(-x) - \dfrac{\csc^2 x}{\cot^2 x}$