Graphing Secant, Cosecant, Tangent, and Cotangent Functions

Remember, $\sec x = \dfrac{1}{\cos x}$ and $\csc x = \dfrac{1}{\sin x}$.

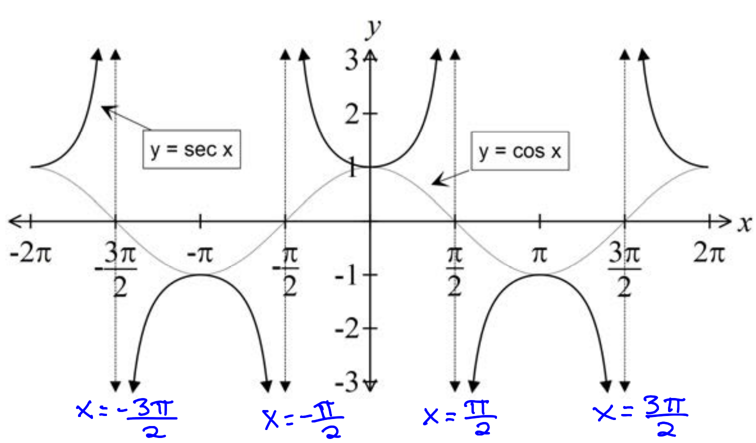
- We aren't allowed to divide by 0. This means:
  - Whenever $\cos x = 0$, $\sec x$ is undefined, and whenever $\sin x = 0$, $\csc x$ is undefined.
    - Places where $\cos x = 0$ and $\sec x$ is undefined: $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$, etc.
    - Places where $\sin x = 0$ and $\csc x$ is undefined: $0, \pi, 2\pi, 3\pi, 4\pi$, etc.
  - The graphs of $y = \sec x$ and $y = \csc x$ have vertical asymptotes at these locations.
  - **To Find the Equations of the Asymptotes:**
    - Start with any x-value where the function is undefined.
      Add this value to k times the distance between the asymptotes.
      ★ $x = \text{asymptote} + (\text{distance between asymptotes}) \cdot k$

Graphing Secant Functions

- To graph $y = a\sec\left[b(x-c)\right]+d$:
  - Sketch the graph of $y = a\cos\left[b(x-c)\right]+d$.
  - Wherever the graph of the cosine function crosses its center point, draw a vertical asymptote.
  - The local maxima of the graph of the cosine function become local minima on the graph of the secant function with $y \to \infty$ as x approaches the asymptotes on either side. The local minima of the graph of the cosine function become local maxima on the graph of the secant function with $y \to -\infty$ as x approaches the asymptotes on either side.

Key points on the graph of $y = \sec x$:

x	0	$\pi/2$	π	$3\pi/2$	2π
$y = \sec x$	1	undef.	–1	undef.	1



Asymptotes: $x = \frac{\pi}{2} + \pi k$, where k is any integer

any one of the asymptotes

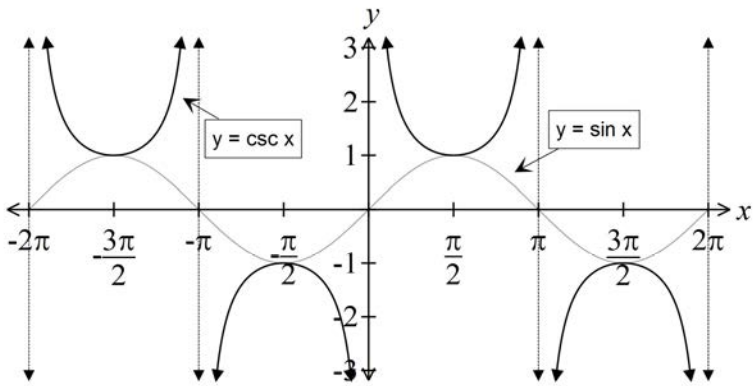
distance between asymptotes

Graphing Cosecant Functions

- To graph $y = a\csc\left[b(x-c)\right]+d$:
    - Sketch the graph of $y = a\sin\left[b(x-c)\right]+d$.
    - Wherever the graph of the sine function crosses its center point, draw a vertical asymptote.
    - The local maxima of the graph of the sine function become local minima on the graph of the cosecant function with $y \to \infty$ as x approaches the asymptotes on either side. The local minima of the graph of the sine function become local maxima on the graph of the cosecant function with $y \to -\infty$ as x approaches the asymptotes on either side.

Key points on the graph of $y = \csc x$:

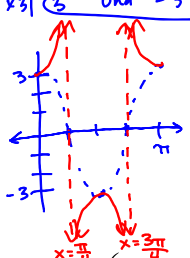
x	0	$\pi/2$	π	$3\pi/2$	2π
$y=\csc x$	undef.	1	undef.	-1	undef.



Examples: Graph the following functions. Find the period, asymptotes, and range of each.

a) $y = 3\sec(2x)$

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\div 2$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	1	und	-1	und	1
$\times 3$	3	und	-3	und	3



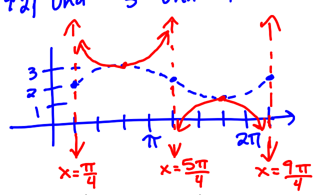
period $= \frac{2\pi}{2} = \pi$

asymptotes: $x = \frac{\pi}{4} + \frac{\pi}{2}k$

range: $(-\infty,-3]\cup[3,\infty)$

b) $y = \csc\left(x - \frac{\pi}{4}\right)+2$

	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\to\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{3\pi}{4}$	$\frac{5\pi}{4}$	$\frac{7\pi}{4}$	$\frac{9\pi}{4}$
	und	1	und	-1	und
$+2$	und	3	und	1	und



period: 2π

asymptotes: $x = \frac{\pi}{4} + \pi k$

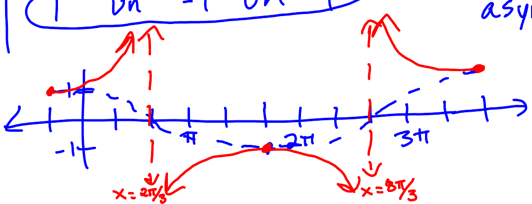
range: $(-\infty,1]\cup[3,\infty)$

c) $y = \sec\left(\frac{1}{2}x + \frac{\pi}{6}\right)$

$\sec\left[\frac{1}{2}\left(x+\frac{\pi}{3}\right)\right]$

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\div\frac{1}{2}$	0	π	2π	3π	4π
$-\frac{\pi}{3}$	$-\frac{\pi}{3}$	$\frac{2\pi}{3}$	$\frac{5\pi}{3}$	$\frac{8\pi}{3}$	$\frac{11\pi}{3}$
y	1	un	-1	un	1

period $= \frac{2\pi}{1/2} = 4\pi$



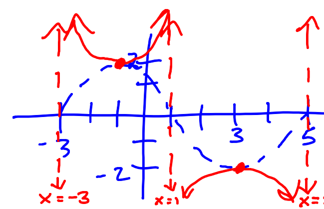
asymptotes: $x = \frac{2\pi}{3} + 2\pi k$

range: $(-\infty,-1]\cup[1,\infty)$

d) $y = 2\csc\left(\frac{\pi}{4}x + \frac{3\pi}{4}\right)$

$y = 2\csc\left[\frac{\pi}{4}(x+3)\right]$

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\div\frac{\pi}{4}$	0	2	4	6	8
-3	-3	-1	1	3	5
y	un	2	un	-2	un

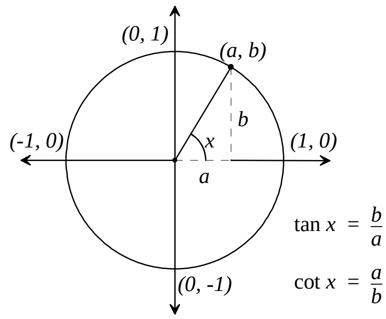


period $= \frac{2\pi}{\pi/4} = 8$

asymptotes: $x = 1 + 4k$

range: $(-\infty,-2]\cup[2,\infty)$

Let (a,b) be coordinates of points on the unit circle. For any given angle x, $\tan x = b/a$. This means that $y = \tan x$ is undefined whenever $a = 0$. For any given angle x, $\cot x = a/b$. This means that $y = \cot x$ is undefined whenever $b = 0$. Notice that it takes π radians for the values of the tangent and cotangent to make one complete cycle.

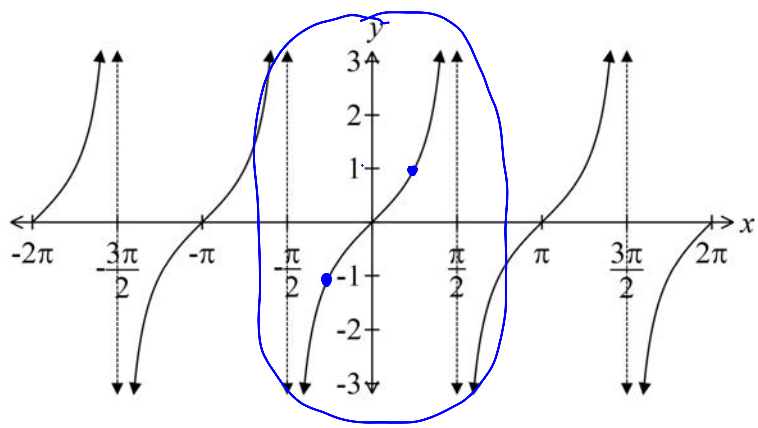


Graphing Tangent Functions:

The domain of $y = \tan x$ is the set of all real numbers except numbers of the form $\pi/2 + k\pi$, where k is an integer. The equations of the vertical asymptotes are $x = \pi/2 + k\pi$, where k is an integer.

Key points on the graph of $y = \tan x$:

x	$-\pi/2$	$-\pi/4$	0	$\pi/4$	$\pi/2$
$y = \tan x$	undef.	-1	0	1	undef.



To graph $y = a\tan\left[b(x-c)\right] + d$:

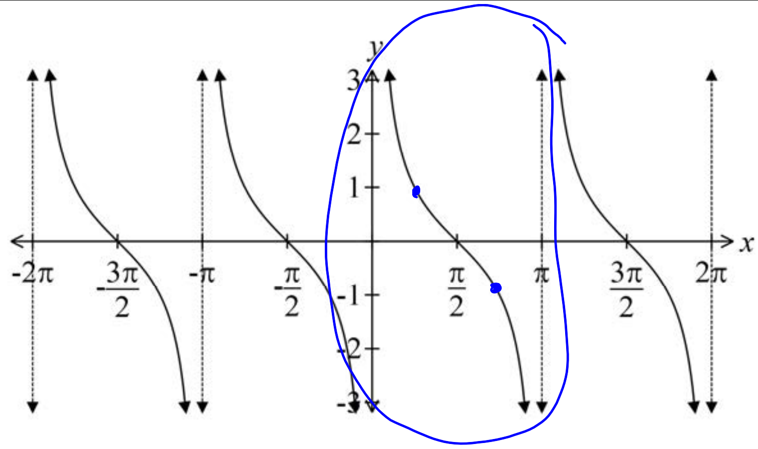
1. Start with the three key points on the graph of $y = \tan x$ and the equations of the asymptotes.
2. Find three key points and the asymptotes for $y = a\tan\left[b(x-c)\right] + d$ by:
   a. dividing each *x*-coordinate by *b* and adding *c*. (Treat the equations of the asymptotes like *x*-coordinates.)
   b. multiplying each *y*-coordinate by *a* and adding *d*.
3. Sketch one cycle of $y = a\tan\left[b(x-c)\right] + d$ through the three new points and approaching the new asymptotes.

★ **The period of $y = a\tan\left[b(x-c)\right] + d$ and $y = a\cot\left[b(x-c)\right] + d$ is $\boxed{\pi/b}$ rather than $2\pi/b$.**

Graphing Cotangent Functions:
The domain of $y = \cot x$ is the set of all real numbers except numbers of the form $k\pi$, where k is an integer.
The equations of the vertical asymptotes are $x = k\pi$, where k is an integer.

Key points on the graph of $y = \cot x$:

x	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
$y = \cot x$	undef.	1	0	-1	undef.



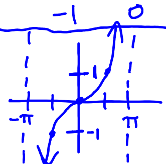
To graph $y = a\cot\left[b(x-c)\right]+d$:

1. Start with the three key points on the graph of $y = \cot x$ and the equations of the asymptotes.
2. Find three key points and the asymptotes for $y = a\cot\left[b(x-c)\right]+d$ by:
   a. dividing each x-coordinate by b and adding c. (Treat the equations of the asymptotes like x-coordinates.)
   b. multiplying each y-coordinate by a and adding d.
3. Sketch one cycle of $y = a\cot\left[b(x-c)\right]+d$ through the three new points and approaching the new asymptotes.

Examples: Graph the following functions. Find the period and the equations of the asymptotes of each.

$y = \tan\left(\frac{1}{2}x\right)$

x	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
$\div\frac{1}{2}$	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π
y	und	-1	0	1	und

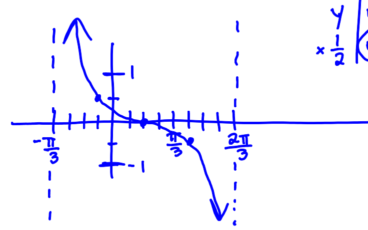


period $= \frac{\pi}{1/2} = 2\pi$

asymptotes: $x = \pi + 2\pi k$

$y = \frac{1}{2}\cot\left(x+\frac{\pi}{3}\right)$

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$-\frac{\pi}{3}$	$-\frac{4\pi}{12}$	$-\frac{\pi}{12}$	$\frac{2\pi}{12}$	$\frac{5\pi}{12}$	$\frac{8\pi}{12}$
y	un	1	0	-1	un
$\times\frac{1}{2}$	un	$\frac{1}{2}$	0	$-\frac{1}{2}$	un



period $= \pi$

asymptotes:

$x = \frac{2\pi}{3} + \pi k$

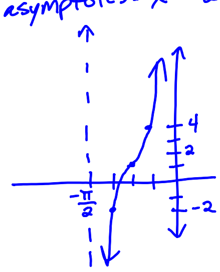
$y = 3\tan\left(2x+\frac{\pi}{2}\right)+1$

$y = 3\tan\left[2\left(x+\frac{\pi}{4}\right)\right]+1$

period $= \frac{\pi}{2}$

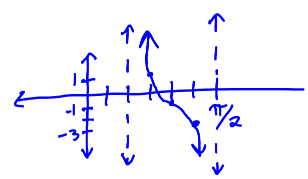
asymptotes: $x = \frac{\pi}{2}k$

x	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
$\div 2$	$-\frac{\pi}{4}$	$-\frac{\pi}{8}$	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$
$-\frac{\pi}{4}$	$-\frac{4\pi}{8}$	$-\frac{3\pi}{8}$	$-\frac{2\pi}{8}$	$-\frac{\pi}{8}$	0
y	un	-1	0	1	un
$\times 3$	un	-3	0	3	un
$+1$	un	-2	1	4	un



$y = 2\cot\left[3\left(x-\frac{\pi}{6}\right)\right]-1$

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$\div 3$	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$+\frac{\pi}{6}$	$\frac{2\pi}{12}$	$\frac{3\pi}{12}$	$\frac{4\pi}{12}$	$\frac{5\pi}{12}$	$\frac{6\pi}{12}$
y	und	1	0	-1	und
$\times 2$	und	2	0	-2	und
-1	und	1	-1	-3	und



period $= \pi/3$

asymptotes: $x = \pi/6 + \pi/3\,k$