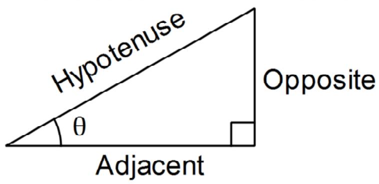
Right Triangle Trigonometry

Trigonometric Ratio: A ratio of the lengths of two sides of a right triangle. The three main trigonometric rations are sine (sin), cosine (cos), and tangent (tan).

★ If θ is an acute angle of a right triangle, then the values of the sine, cosine, and tangent of θ are defined in terms of the lengths of the three sides of the triangle – the leg adjacent (next to) θ, the leg opposite (across from) θ, and the hypotenuse – as follows:



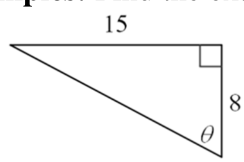
$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

A common way to remember this is SOH-CAH-TOA

★ **Remember:** For all right triangles, you can use the Pythagorean Theorem: $\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$

Examples: Find the exact values of $\sin\theta$, $\cos\theta$, and $\tan\theta$.

a)



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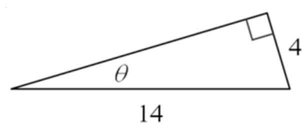
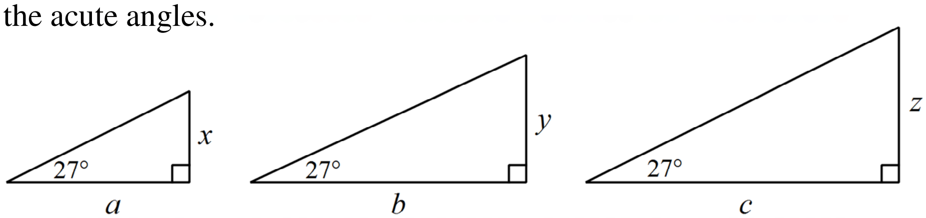


Examples: Draw and label a triangle, find the length of the missing side, and find the requested values.

a) Find $\sin\theta$ and $\tan\theta$ if $\cos\theta = \dfrac{9}{41}$

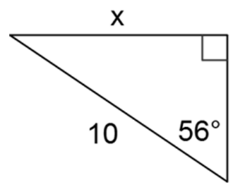
b) Find $\sin\theta$ and $\cos\theta$ if $\tan\theta = \dfrac{35}{12}$

No matter how big the triangle is, the values of the trigonometric functions for a certain size angle will remain the same. For example, in the diagram below, $\tan 27° = x/a = y/b = z/c$. The value of tan 27° never changes, no matter what size the triangle is. In fact, tan 27° ≈ 0.5095254495… Sine and cosine act in a similar way. The sine or cosine of a certain angle is the same no matter what size the triangle is. This allows us to use the value of trigonometric functions to find the length of a side of a right triangle if we know the length of another side and the size of one of the acute angles.



Examples: Write an equation involving sine, cosine, or tangent that can be used to find the missing length. Then solve the equation. Round your answers to the nearest tenth.

a)



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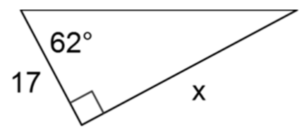

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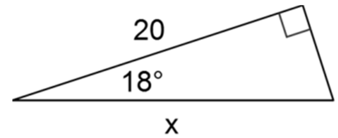

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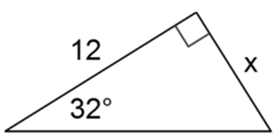

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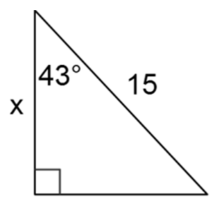

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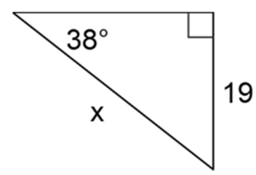


Inverse Sine, Cosine, and Tangent Functions

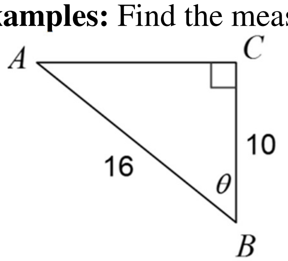
- The inverse sine or arcsine of x ($\sin^{-1} x$ or $\arcsin x$) is the angle between $-90°$ and $90°$ whose sine is x.
  - If $\sin\theta = x$, and $-90° \le \theta \le 90°$, then $\theta = \sin^{-1} x$.
- The inverse cosine or arccosine of x ($\cos^{-1} x$ or $\arccos x$) is the angle between $0°$ and $180°$ whose cosine is x.
  - If $\cos\theta = x$, and $0° \le \theta \le 180°$, then $\theta = \cos^{-1} x$.
- The inverse tangent or arctangent of x ($\tan^{-1} x$ or $\arctan x$) is the angle between $-90°$ and $90°$ whose tangent is x.
  - If $\tan\theta = x$, and $-90° < \theta < 90°$, then $\theta = \tan^{-1} x$.

★ **The -1 in $\sin^{-1} x$ does not indicate a reciprocal! $\sin^{-1} x \ne 1/\sin x$.**
$\sin^{-1} x$, $\cos^{-1} x$, and $\tan^{-1} x$ are angles!

★ **Use inverse functions when you know the sine, cosine, or tangent of an angle and want to know how big the angle is.**

Examples: Find the measure of the indicated angle to the nearest tenth of a degree.

a)



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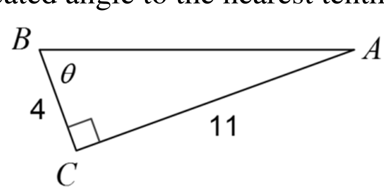

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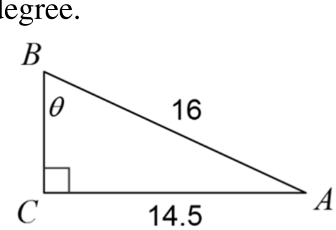
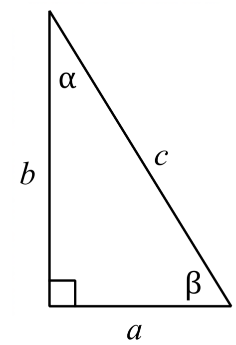


Solving Right Triangles

Finding all the missing angle measures and side lengths of a triangle is called "solving a triangle". In a right triangle, we often name the acute angles α (alpha) and β (beta) and the lengths of the sides opposite those angles a and b, respectively. The right angle is sometimes called γ (gamma) and the length of the side opposite the right angle (the hypotenuse) is c.



- ★ If you know the lengths of two of the sides and want the length of the third side, use the Pythagorean Theorem.
- ★ If you know the measure of one of the acute angles and want the measure of the other acute angle, use the fact that the angles in a triangle add to 180° (so the acute angles in a right triangle add to 90°).
- ★ If you know the measure of one angle and the length of one side and want the lengths of the other sides, use sin, cos, or tan.
- ★ If you know the lengths of the sides and want to figure out the angle measures, use inverse functions ($\sin^{-1}$, $\cos^{-1}$, or $\tan^{-1}$).

Examples:

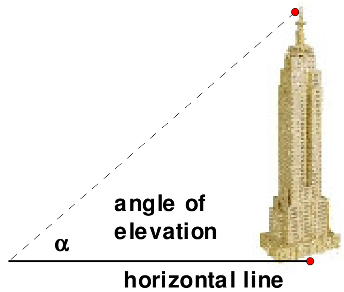
Solve the right triangle in which $\alpha = 30°$ and $c = 6$.

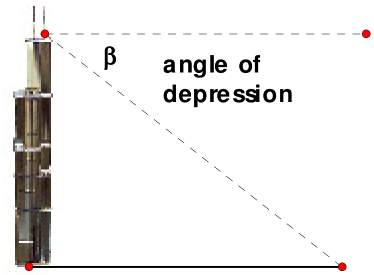
Solve the right triangle in which $a = 2$ and $b = 5$.

Solve the right triangle in which $\beta = 20°$ and $b = 15$.

Solve the right triangle in which $a = 5$ and $c = 13$.

Using trigonometry, we can find the size of an object without actually measuring the object. Two common terms used in this regard are **angle of elevation** and **angle of depression**.





Examples:

The angle of elevation of the top of a cell phone tower is 38.2° at a distance of 344 feet from the tower. What is the height of the tower?

At one location, the angle of elevation of the top of an antenna is 44.2°. At a point that is 100 feet closer to the antenna, the angle of elevation is 63.1°. What is the height of the antenna?