Implicit Differentiation

Look at figure 4.7
What is the equation of this function?
Does this graph have tangents?
How can we differentiate this function?
On this particular example we would have to break it into three function pieces. These 3 pieces are not explicitly stated, they are defined implicitly. Therefore the process by which we find the derivative of such a function is called **Implicit differentiation**.
Implicit differentiation is used when y cannot be written explicitly as a function of x.
What does implicit and explicit mean?

Implicit differentiation – treat y as a differentiable function of *x* and apply the rules of differentiation.

Let's start with a surprisingly simple concept. What is the derivative of y with respect to x?
Answer: The derivative of y is dy/dx.

Example:

$$Find\ \frac{dy}{dx}\ if\ \ y^2 = x$$

$$Use\ \ the\ \ chain\ \ rule:$$

$$\frac{d}{dx} outside\, of\ \ inside\, times\, deriv\, inside\ \ 2y\frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{2y}$$

dy/dx does not always have to be a function of x. Sometimes it is useful to have it be a function of y. Let's read the paragraph under Example 1.

Implicit Differentiation Process:

1. Differentiate both sides of the equation with respect to x.
2. Collect the terms with dy/dx on one side of the equation.
3. Factor out dy/dx.
4. Solve for dy/dx.

Examples:

$$\frac{dy}{dx} \qquad if \qquad 2x^3 - 3y^2 = 8$$

$\frac{d}{dx} \quad if \quad x^2 + xy - y^2 = 1$

#18. $\frac{d}{dx} \quad if \quad x^2 + y^2 = 25$

Find the lines that are (a) tangent and (b) normal to the curve at the given point. (3,-4)

We can also find higher order derivatives implicitly.

Example: $Find\ \frac{d^2y}{dx^2} \quad of \quad 2x^3 - 3y^2 = 8$

Rational Powers of Differentiable Functions: We can apply the **power rule**,

As we have shown before: $\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$

$\sqrt{x} = x^{\frac{1}{2}}$

$\frac{d}{dx} x^{\frac{1}{2}} \Rightarrow \frac{1}{2} x^{\frac{-1}{2}} = \frac{1}{2x^{\frac{1}{2}}} = \frac{1}{2\sqrt{x}}$

Something more challenging?!

$Find\ \frac{dy}{dx} \quad if \quad y = (2x-3)^{\frac{2}{3}}$

$Find\ \frac{dy}{dx} \quad if \quad y = \sqrt[5]{\sin(x)}$

Exploration 1