Angular and Linear Velocity

Velocity: The rate at which the location of an object is changing with respect to time.

Angular Velocity: The rate at which the central angle is changing for an object moving in a circle. If a point is in motion on a circle through an angle of α radians in time t, then its angular velocity ω is given by $\omega = \frac{\alpha}{t}$. Angular velocity is usually expressed as radians per unit of time (radians/hr, radians/min, radians/sec, etc.)

Examples:
Convert 650 rpm (revolutions per minute) to radians per minute.
(Use the fact that 1 revolution = 2π radians.)

$$\frac{650 \text{ rev}}{\text{min}} \frac{2\pi \text{ rad}}{\text{rev}} = 1300\pi \text{ rad/min} \approx 4084.1 \text{ rad/min}$$

Convert the angular velocity of 1600 rad/hr to rev/hr.

$$\frac{1600 \text{ rad}}{\text{hr}} \frac{\text{rev}}{2\pi \text{ rad}} \approx 254.6 \text{ rev/hr}$$

A 24-inch lawnmower blade rotates at a rate of 2000 rpm. What is the angular velocity in radians per second of a point on the tip of the blade?

$$\frac{2000 \text{ rev}}{\text{min}} \frac{2\pi \text{ rad}}{\text{rev}} \frac{\text{min}}{60 \text{ sec}} \approx 209.4 \text{ rad/sec}$$

A particle is moving in a circular path with a radius of 9 ft. at 30 radians per minutes. How fast is the particle rotating in revolutions per second?

$$\frac{30 \text{ rad}}{\text{min}} \frac{\text{rev}}{2\pi \text{ rad}} \frac{\text{min}}{60 \text{ sec}} \approx 0.08 \text{ rev/sec}$$

Linear Velocity: The rate at which the position of the object is changing with respect to time. If a point is in motion on a circle of radius r through an angle of α radians in time t, then its linear velocity v is given by $v = \frac{s}{t}$, where s is the arc length determined by $s = \alpha r$.

1 rev = 1 circumference = $2\pi r$ or πd

1 radian = 1 radius

Examples:

A propeller with a radius of 1.6 meters is rotating at 1500 revolutions per minute. What is the linear velocity in meters per second for a point on the tip of the propeller?

$C = 2\pi(1.6) = 3.2\pi$ m $\leftarrow$ 1 rev

$$\frac{1500 \text{ rev}}{\text{min}} \cdot \frac{3.2\pi \text{ m}}{\text{rev}} \cdot \frac{\text{min}}{60 \text{ sec}} \approx \boxed{251.3 \text{ m/sec}}$$

Find the angular velocity in radians per second for a particle that is moving along a circle with diameter 15 meters at a linear velocity of 20 meters per second.

$r = 7.5$ m = 1 rad

$C = 15\pi$ m = 1 rev

$$\frac{20 \text{ m}}{\text{sec}} \cdot \frac{\text{rad}}{7.5 \text{ m}} \approx \boxed{2.7 \text{ rad/sec}}$$

OR

$$\frac{20 \text{ m}}{\text{sec}} \cdot \frac{\text{rev}}{15\pi \text{ m}} \cdot \frac{2\pi \text{ rad}}{\text{rev}} \approx \boxed{2.7 \text{ rad/sec}}$$

What is the linear velocity in miles per hour of the tip of a 20-inch lawnmower blade that is rotating at 3000 rpms?

20-inch ← diameter

$C = 20\pi$ in = 1 rev

$$\frac{3000 \text{ rev}}{\text{min}} \cdot \frac{20\pi \text{ in}}{\text{rev}} \cdot \frac{\text{ft}}{12 \text{ in}} \cdot \frac{\text{mi}}{5280 \text{ ft}} \cdot \frac{60 \text{ min}}{\text{hr}} \approx \boxed{178.5 \text{ mph}}$$

Find the angular velocity in radians per minute for a particle that is moving in a circular path at 95 mph on a circle with a radius of 8 inches.

$r = 8$ in = 1 rad

$C = 2\pi(8) = 16\pi$ in = 1 rev

$$\frac{95 \text{ mi}}{\text{hr}} \cdot \frac{5280 \text{ ft}}{\text{mi}} \cdot \frac{12 \text{ in}}{\text{ft}} \cdot \frac{\text{rad}}{8 \text{ in}} \cdot \frac{\text{hr}}{60 \text{ min}} = \boxed{12{,}540 \text{ rad/min}}$$

OR

$$\frac{95 \text{ mi}}{\text{hr}} \cdot \frac{5280 \text{ ft}}{\text{mi}} \cdot \frac{12 \text{ in}}{\text{ft}} \cdot \frac{\text{rev}}{16\pi \text{ in}} \cdot \frac{2\pi \text{ rad}}{\text{rev}} \cdot \frac{\text{hr}}{60 \text{ min}} = \boxed{12{,}540 \text{ rad/min}}$$

Linear Velocity in Terms of Angular Velocity: If v is the linear velocity of a point on a circle of radius r, and ω is its angular velocity, then $v = r\omega$.

Example:

Any point on the surface of the earth (except at the poles) makes one revolution (2π radians) about the axis of the earth in 24 hours. So the angular velocity of a point on the earth is $2\pi/24$ or $\pi/12$ radians per hour. The linear velocity of a point on the surface of the earth depends on its distance from the axis of the earth. What is the linear velocity in miles per hour of a point on the equator? (Use 3950 miles as the radius of the earth).

$C = 2\pi(3950) = 7900\pi$ miles

$$\frac{1 \text{ rev}}{24 \text{ hrs}} \cdot \frac{7900\pi \text{ mi}}{\text{rev}} \approx \boxed{1034.1 \text{ mi/hr}}$$

OR

$$\frac{\pi \text{ rad}}{12 \text{ hr}} \cdot \frac{3950 \text{ mi}}{\text{rad}} \approx \boxed{1034.1 \text{ mi/hr}}$$