Angular and Linear Velocity

Velocity: The rate at which the location of an object is changing with respect to time.

Angular Velocity: The rate at which the central angle is changing for an object moving in a circle. If a point is in motion on a circle through an angle of α radians in time *t*, then its angular velocity ω is given

by $\omega = \frac{\alpha}{t}$. Angular velocity is usually expressed as radians per unit of time (radians/hr, radians/min, radians/sec, etc.)

Examples:

Convert 650 rpm (revolutions per minute) to radians per minute. (Use the fact that 1 revolution = 2π radians).

Convert the angular velocity of 1600 rad/hr to rev/hr.

A 24-inch lawnmower blade rotates at a rate of 2000 rpm. What is the angular velocity in radians per second of a point on the tip of the blade?

A particle is moving in a circular path with a radius of 9 ft. at 30 radians per minute. How fast is the particle rotating in revolutions per second?

Linear Velocity: The rate at which the position of the object is changing with respect to time. If a point is in motion on a circle of radius *r* through an angle of α radians in time *t*, then its linear velocity *v* is given by $v = \frac{s}{t}$, where *s* is the arc length determined by $s = \alpha r$.

Examples:

A propeller with a radius of 1.6 meters is rotating at 1500 revolutions per minute. What is the linear velocity in meters per second for a point on the tip of the propeller?

Find the angular velocity in radians per second for a particle that is moving along a circle with diameter 15 meters at a linear velocity of 20 meters per second.

What is the linear velocity in miles per hour of the tip of a 20-inch lawnmower blade that is rotating at 3000 rpms?

Find the angular velocity in radians per minute for a particle that is moving in a circular path at 95 mph on a circle with a radius of 8 inches.

Linear Velocity in Terms of Angular Velocity: If *v* is the linear velocity of a point on a circle of radius *r*, and ω is its angular velocity, then $v = r\omega$.

Example:

Any point on the surface of the earth (except at the poles) makes one revolution $(2\pi \text{ radians})$ about the axis of the earth in 24 hours. So the angular velocity of a point on the earth is $2\pi/24$ or $\pi/12$ radians per hour. The linear velocity of a point on the surface of the earth depends on its distance from the axis of the earth. What is the linear velocity in miles per hour of a point on the equator? (Use 3950 miles as the radius of the earth).