Velocity and Other Rates of Change

In general, ***using the difference quotient*** *and* ***taking the limit as h→0*** will give us the ***derivative***, a formula for the ***tangent to the curve***, or the ***instantaneous rate of change.*** Knowing what the rate of change is at any given point in time can be very useful in medicine, in factory production, in economics, and so forth.

Recall, when we were interested at a specific point $x = a$ we found $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$. Even if the function doesn't relate to time, we still say ***instantaneous rate of change***. In fact, whenever we say rate of change it is understood ***from now on*** that we mean instantaneous rate of change.

Example: Find the rate of change of the area of a circle with respect to the radius.
How do we write this?
The change in y with respect to x was written $\frac{\Delta y}{\Delta x} \, or \, \frac{dy}{dx}$.
Here it is area (A) with respect to radius r, so we need to write it $\frac{dA}{dr}$. We need a function that relates the two. What is the formula for finding the area of a circle? _________What is its derivative? __________

Evaluate the rate of change for r = 5 and r = 10

What units would be appropriate for $\frac{dA}{dr}$?

Motion along a Line

Linear motion, position graphs s or s(t), velocity graph v(t) or s′, acceleration graph a(t) or s″
Suppose we have a position function that tells us the position of an object with respect to time: s=f(t)
Example: $s(t) = t^2 - 3t$

The ***displacement*** of the object over the time interval from t to t+Δt, or net change in position is Δs = f(t+Δt)-f(t) and the ***average velocity*** of the object over that time interval is
$v_{av} = \frac{displacement}{travel\ \ time} = \frac{\Delta s}{\Delta t} = \frac{f(t+\Delta t) - f(t)}{\Delta t}$ Does this formula look familiar?

Instantaneous velocity is the derivative of the position function s=f(t) with respect to time. At time t the velocity is $v(t) = \frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{f(t+\Delta t) - f(t)}{\Delta t}$

In our example position function of $s(t) = t^2 - 3t$ find the derivative or in other words the velocity. _______
What is the velocity at t=3? _______

Example 2

Back to $s(t) = t^2 - 3t$ Find velocity at t=1.

Sometimes velocity is positive and sometimes velocity is negative. Velocity not only tells us how fast, it also tells us which direction.

Speed is the absolute value of velocity. Speed = | v(t)| = | ds/dt|
Example 3

If we take the 2^{nd} derivative of the position, we get the acceleration or rate of change of the velocity. The rate at which a body's velocity changes is called the body's acceleration. The acceleration measures how quickly the body picks up or loses speed.

Definition: Acceleration
Acceleration is the ***derivative of velocity with respect to time***. If a body's velocity at time t is $v(t) = ds/dt$, then the body's acceleration at time t is $a(t) = dv/dt = d^2s/dt^2$.

Remember Free-fall constants (on Earth)

English units:	(s in feet)	$s = 16t^2$	$g = a = 32$ ft/sec^2
Metric units:	(s in meters)	$s = 4.9t^2$	$g = a = 9.8$ m/sec^2

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Example 4 & 5

Exploration #2 and #3

The first derivative allows us to see how sensitive a function is to change. If d/dx is constant, it's not sensitive. If it's not, it can have varying degrees of sensitivity.

When a small change in x produces a large change in the value of a function we say that the function $f(x)$ is relatively sensitive to changes in x. The derivative $f'(x)$ is a measure of this sensitivity.

In other words the steeper the curve the more sensitive to change the function is.
Example 6

Derivatives in Economics

As we have seen $s' = velocity = \dfrac{ds}{dt}$ $\quad and \quad$ $s'' = v' = acceleration = \dfrac{d^2s}{dt^2}$

The derivative of a cost function gives us the marginal cost, the rate at which the cost changes as production amount changes.

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