Differentiability

In order for a derivative to exist at a point, the left and right derivatives at that point must exist and they must be equal. The derivative is the rate of change, $\frac{dy}{dx}$, or the slope of the tangent at that point. Putting these two things together, can we produce a graph of a function that does not have a derivative at "a".

A function with **a corner** (slope from right ≠ slope from left example: $|x|$)

A function with **a cusp** (slopes of the secant lines approach ∞from one side and -∞ from the other side. Example: $f(x) = x^{\frac{2}{3}}$)

A **vertical tangent** (slopes of the secant lines approach either ∞ or -∞ from both sides. Example: $f(x) = \sqrt[3]{x}$)

A **discontinuity** (will cause one or both of the one-sided derivatives to be nonexistent. Example: Unit step function)

Differentiable functions have graphs that are continuous and smooth. So, if we find points of discontinuity or corners then it will not be **differentiable at that point**. The exception to that is if we have a vertical tangent, it's smooth, but undifferentiable at that point.

Example: $f(x) = |x+2| - 1$ Where is this function not differentiable?

Most functions we use are differentiable at points in their domain. They will be smooth continuous curves with well-defined slopes. They will not have corners, cusps, vertical tangent lines, or points of discontinuity with their domains. Polynomials are differentiable, as are rational, trigonometric, exponential, and logarithmic functions. Composites of differentiable functions are differentiable, and so are sums, products, integer powers, and quotients of differentiable functions, where defined.

If we take a point on a smooth continuous curve, (what does this mean?______________) and we zoom in on it, what will it look like? ______________

Differentiable functions are said to be locally linear or to have local linearity. Wouldn't it just be nice to have your calculator figure all this out for you? To some extent they do. It could use the **difference quotient**, $\frac{f(a+h) - f(a)}{h}$ but a better approximation of the derivative, slope at a point is the **symmetric difference quotient** $\frac{f(a+h) - f(a-h)}{2h}$

Your calculator uses a numerical derivative of *f* at a point *a*.
NDER or nDeriv (f(x), x, a) Ti-84 math #8 nDeriv (

Examples:

$f(x) = x^2 \;\; at\; x = 2$

$f(x) = x^3 \;\; at\; x = 2$

$f(x) = x^2 + 4x \;\; at\; x = 1$

$f(x) = |x| \;\; at\; x = 0$

$f(x) = x^{\frac{2}{3}} \;\; at\; x = 0$

$f(x) = x^{\frac{1}{3}} \;\; at\; x = 0$

Be assured that since they allow calculators on AP tests, there are questions designed to see if you rely too heavily on your calculator. Understanding is the key!

Example: 4
Graph:

$y_1 = \ln x$

$y_2 = nderiv(y_1, x, x)$

Differentiability implies Continuity
If a function *f* has a derivative at x = a then the function *f* is continuous at x = a
To prove this we show that

$$\lim_{x\to a} f(x) = f(a)$$

$$or\;\; \lim_{x\to a} f(x) - f(a) = 0$$

$$\lim_{x\to a} f(x) - f(a) = \lim_{x\to a}(x-a)\frac{(f(x) - f(a))}{x-a}$$

$$= \lim_{x\to a}(x-a)\cdot\lim_{x\to a}\frac{(f(x) - f(a))}{x-a}$$

$$= 0\cdot f'(x) \qquad f'\; must\; exist$$

$$= 0$$

$$then\; \lim_{x\to a} f(x) = f(a)\;\; and\;\; therefore\;\; continuous$$

Theorem 2 Intermediate Value Theorem for Derivatives: If **a** and *b* are any two points in an interval on which *f* is differentiable, then *f'* takes on every value between *f'(a)* and *f'(b).*

Does any function have the unit step function as its derivative? "No" Let a<0 and b>0. U(a) =-1 and U(b) =1, but U does not take on any value between -1 and 1.