Derivatives of a Function

In chapter 2 we discussed rates of change. Rates of change are a huge branch of mathematics called differential calculus. Remember when we found the slope of the tangent line at any point "a" of the function f(x) = x^2+4x to be 2a+4….this was the first look at what we call the derivative of a function.

Derivatives allow us to describe how things change. Calculus is mathematics of motion or the math that describes things that change. Derivatives are the first step to describing complex motion/change problems. Before we can solve complex problems, we must understand how derivatives work.

The derivative of a function *f*(*x*) is called *f ′(x)*. "*f* prime of *x*"

f ′ (x) is defined by $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ provided the limit exists.

If *f ′* exists *f* has a derivative (is differentiable) at *x*.

The domain of *f ′* may be smaller than the domain of *f*. A differentiable function has a derivative at each point in its domain.

Consider $f(x) = x^3$ What is $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$?

An alternate definition of the derivative can be found by using points x and *a* and finding the limit of the slope of the secant line as x→a instead of using h.

$$f'(x) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Find f ′ of $f(x) = x^3$ using this alternate definition

$$f'(x) = \lim_{x \to a} \frac{x^3 - a^3}{x - a} = \frac{(x-a)(x^2 + ax + a^2)}{x - a} = \lim_{x \to a} x^2 + ax + a^2 = a^2 + a \cdot a + a^2 = 3a^2$$

Work in groups to find *f ′*(x) when f (x) = $\sqrt{x}$

Derivatives are another tool that helps us describe a function and its graph. They tell us the slope of the curve at any point.

Recall y = f (x)

f ′ "f prime" or the "derivative of *f* "

y′ "y prime" or the "derivative of y"

These are brief, but do not name the independent variable.

$\frac{dy}{dx}$ "dy dx" ("the derivative of y with respect to x") Names both variables & used d for derivative.

$\frac{df}{dx}$ "df dx" ("the derivative of *f* with respect to x") Emphasizes the function's name.

$\frac{d}{dx}f(x)$ "d dx of f at x" ("the derivative of *f* at x") Emphasizes the idea that differentiation is an operation performed on *f*. (the slope of the tangent at x)

The reason we have so many types of notation is that Newton and Leibniz developed differential calculus, each on their own, hundreds if not thousands of miles apart.

This next idea is extremely important to your understanding of calculus. The AP test focuses more and more on understanding and interpretation rather than "plug and chug"

Remember f ′(x) is the derivative of f (x) if we can graph f, why can't we graph f ′?

What is the relationship between the graphs of f(x) and f ′(x)?

Examine Example 3

Given a graph of f (x) approximate the graph of f ′(x). What does f ′(x) mean graphically?

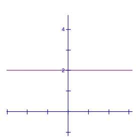
Examine f(x) = x^3 -2x -1

Graph f(x), graph f ′(x), find f ′(x) analytically. Do your graphs look similar?

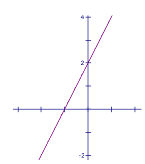
Exploration #1 1-6
#13, #14 page 105

It is also possible to graph the function using the graph of f', but doing this accurately requires what we call an initial condition.
Example 4

Example: f (0) = 3 and f ′(x)



Example: f (-1) =2 and f ′(x)



We can also get the derivative from data. $\frac{dy}{dx}$ means rate of change – change in y with respect to x.

#29 page 107

Right and left hand derivatives
Suppose f (x) is differentiable on [a,b], then it is differentiable at all interior points and right hand derivative exists for a and left hand derivative exists for b.

i.e. $\lim_{h\to 0^+}\frac{f(a+h)-f(a)}{h}$ $\lim_{h\to 0^-}\frac{f(b+h)-f(b)}{h}$

Example 6
In order to be differentiable at a point its Left hand and right hand derivatives must exist and they must be equal.