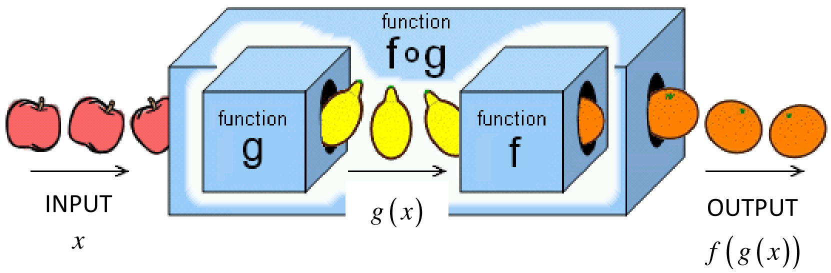
Composite Functions, Inverse Functions

Composite Function: In a composite function, one function is performed, and then a second function is performed on the result of the first function. $(f \circ g)(x) = f(g(x))$ and $(g \circ f)(x) = g(f(x))$.

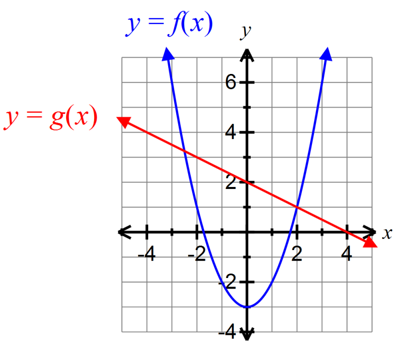


Hints:

- Work inside out. Plug the input into the inside function, then plug the result into the outside function.
- $(f \circ g)(x) = f(g(x))$ is not the same as $(f \cdot g)(x) = f(x) \cdot g(x)$.

Composition of functions — Multiplication of functions

Example: Evaluate each expression using the graph.



a) $(f \circ g)(4)$ b) $(g \circ f)(-1)$

c) $(f \circ f)(1)$ d) $(g \circ g)(0)$

Example: $f(x) = 2x^2$ and $g(x) = 1 - 3x^2$

a) Find $(f \circ g)(4)$ b) Find $(g \circ f)(2)$

c) Find $(f \circ f)(1)$ d) Find $(g \circ g)(0)$

Domain of a Composite Function

The domain of $f \circ g$ is the set of all numbers x in the domain of g such that $g(x)$ is in the domain of f.

Examples: Find the domain of the composite function $(f \circ g)(x)$.

a) $f(x)=\dfrac{1}{x-2},\ g(x)=\sqrt{x}$

b) $f(x)=\dfrac{x}{x-1},\ g(x)=\dfrac{x+5}{x-4}$

Example: $f(x)=x+1$ and $g(x)=x^2+4$

a) Find $(f \circ g)(x)$ and its domain.

b) Find $(g \circ f)(x)$ and its domain.

Example: $f(x)=\dfrac{1}{x+3}$ and $g(x)=-\dfrac{2}{x}$

a) Find $(f \circ g)(x)$ and its domain.

b) Find $(g \circ f)(x)$ and its domain.

c) Find $(f \circ f)(x)$ and its domain.

d) Find $(g \circ g)(x)$ and its domain.

Example: Find functions f and g such that $f \circ g = H$.

a) $H(x)=\left(x^2+1\right)^4$

b) $H(x)=|2x+1|$

One-to-one Function*:** A function is ***one-to-one if for any value of x there is exactly one y (otherwise it wouldn't be a function), and for any value of y, there is exactly one x.

Example: Determine whether the following functions are one to one.

a) $\{(-2,6),(-1,3),(0,2),(1,5),(2,8)\}$ b) $\{(1,1),(2,4),(3,9),(0,0),(-1,1),(-2,4)\}$

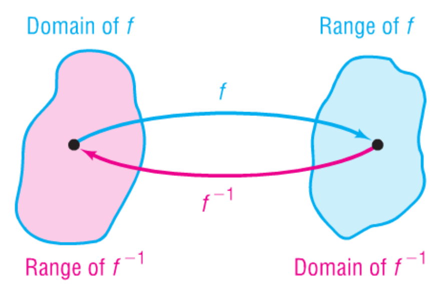
Horizontal Line Test: If every horizontal line intersects the graph of a function f in at most one point, then f is one-to-one.

Example: For each function, use its graph to determine whether the function is one-to-one.

a) $f(x)=|x|$ b) $g(x)=\sqrt[3]{x}$

Theorem: A function that is increasing on an interval I is a one-to-one function on I.
A function that is decreasing on an interval I is a one-to-one function on I.

Inverse Function*:** Two functions are ***inverses if and only if whenever one function contains the element (a,b), the other function contains the element (b,a). If f is a one-to-one function, the correspondence from the range of f back to the domain of f is called the ***inverse function*** of f. The inverse of f is abbreviated f^{-1}.



★ **Domain of f = Range of f^{-1}** **Range of f = Domain of f^{-1}**

Example: Find the inverse of the following one-to-one function: $\{(2,3),(4,5),(6,8),(9,10),(12,14)\}$

If we start with x, apply f, and then apply f^{-1}, we get x back again.
If we start with x, apply f^{-1}, and then apply f, we get x back again.
What f does, f^{-1} undoes, and vice versa. In other words,

$f^{-1}(f(x))=x$, where x is in the domain of f.

$f(f^{-1}(x))=x$ where x is in the domain of f^{-1}.

To verify that two functions are inverses, show that $f(g(x))=x$ and $g(f(x))=x$

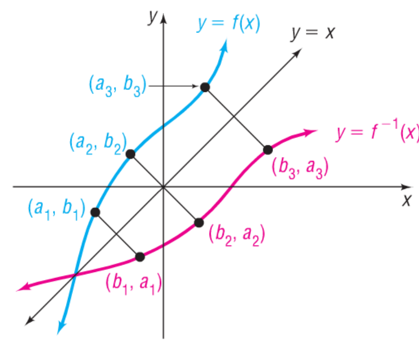
Example: Verify that the functions are inverses.

a) $f(x)=4x;\ g(x)=x/4$ b) $f(x)=4-3x;\ g(x)=\frac{1}{3}(4-x)$

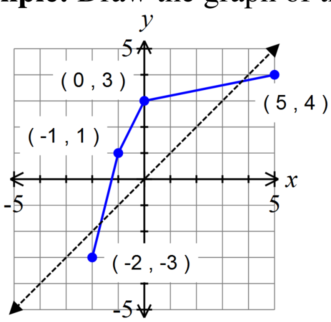
c) $f(x)=\dfrac{2}{x+5};\ g(x)=\dfrac{2}{x}-5$

d) $f(x)=\sqrt[3]{2x};\ g(x)=\dfrac{x^3}{2}$

Theorem: The graph of a function f and the graph of its inverse f^{-1} are symmetric with respect to the line $y=x$.



Example: Draw the graph of the inverse function.



Finding the Inverse of a Function

1. Rewrite $f(x)$ as y in the original equation.
2. Interchange x and y.
3. Solve for y.
4. Replace y with the notation $f^{-1}(x)$.

Example: Find the inverse. State the domain and range of $f(x)$ and the domain and range of $f^{-1}(x)$.

a) $f(x)=-3x+1$

b) $f(x)=\dfrac{2x+3}{5x-4}$