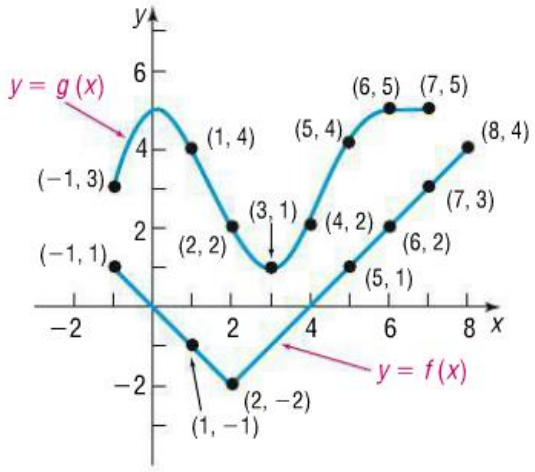
Precalculus

3.1 Functions and Inverse Functions

Name_____ Date_____ Per_____

1. Evaluate each expression using the graphs of $y = f(x)$ and $y = g(x)$ shown in the figure.



a) $(g \circ f)(1)$ b) $(g \circ f)(5)$

c) $(f \circ g)(0)$ d) $(f \circ g)(2)$

e) $(f \circ f)(6)$ f) $(f \circ f)(-1)$

g) $(g \circ g)(7)$ h) $(g \circ g)(-1)$

For problems 2-5, find the following:

a) $(f \circ g)(4)$ b) $(g \circ f)(2)$

c) $(f \circ f)(1)$ d) $(g \circ g)(0)$

2. $f(x) = 3x + 2;\ g(x) = 2x^2 - 1$

3. $f(x) = 4x^2 - 3;\ g(x) = 3 - \frac{1}{2}x^2$

4. $f(x) = \sqrt{x+1};\ g(x) = 3x$

5. $f(x) = |x - 2|;\ g(x) = \frac{3}{x^2 + 2}$

For problems 6-10, find the following:

a) $(f \circ g)(x)$ b) $(g \circ f)(x)$

Also state the domain of each composite function!

6. $f(x) = -x;\ g(x) = 2x - 4$

7. $f(x) = 3x + 1;\ g(x) = x^2$

8. $f(x) = \sqrt{x-2};\ g(x) = 1 - 2x$

9. $f(x) = \dfrac{x}{x-1};\ g(x) = -\dfrac{4}{x}$

10. $f(x) = \dfrac{x-5}{x+1};\ g(x) = \dfrac{x+2}{x-3}$

For problems 11-13, find functions f and g so that $f \circ g = H$.

11. $H(x) = (2x+3)^3$

12. $H(x) = \sqrt{x^2+1}$

13. $H(x) = 5x^2 - 2$

14. The surface area *S* (in square meters) of a hot-air balloon is given by $S(r) = 4\pi r^2$, where r is the radius of the balloon (in meters). If the radius r is increasing with time t (in seconds) according to the formula $r(t) = \frac{2}{3}t^3$, find the surface area S of the balloon as a function of the time t.

For problems 15-16, decide whether the function is one-to-one. If it is one-to-one, write the inverse function and state the domain and range of the function and the domain and range of the inverse.

15. $\{(-2,5),(-1,3),(3,7),(4,12)\}$

16. $\{(2,6),(-3,6),(4,9),(1,10)\}$

For problems 17-20, sketch the graph and use the horizontal line test to determine whether the function is one-to-one.

17. $y = (x-3)^2$

18. $y = \sqrt{x} + 2$

19. $y = -x^3$

20. $y = x(x+1)(x-2)$

For problems 21-25, verify that the functions f and g are inverses of each other by showing that $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$.

21. $f(x) = 4x + 8;\ g(x) = \frac{x}{4} - 2$

22. $f(x) = 3 - 2x;\ g(x) = -\frac{1}{2}(x-3)$

23. $f(x) = x^3 - 8;\ g(x) = \sqrt[3]{x+8}$

24. $f(x) = (x-2)^2,\ x \geq 2;\ g(x) = \sqrt{x} + 2$

25. $f(x)=\dfrac{x-5}{2x+3};\ g(x)=\dfrac{3x+5}{1-2x}$

In problems 26-30, the function f is one-to-one. Find its inverse. State the domain and range of f and the domain and range of f^{-1}.

26. $f(x)=4x-2$

27. $f(x)=x^2+4,\ x\geq 0$

28. $f(x)=\dfrac{4}{x+2}$

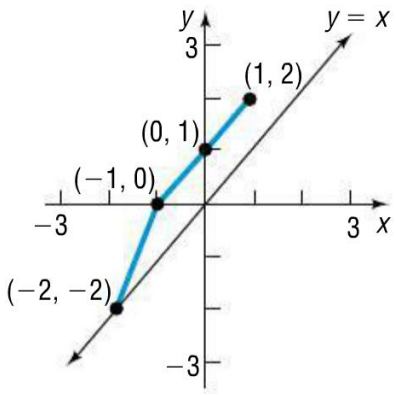
29. $f(x)=\dfrac{2x}{3x-1}$

30. $f(x)=\dfrac{2x-3}{x+4}$

31. $f(x)=\sqrt{x-5}+2$

In problems 31-33, the graph of a one-to-one function f is given. Draw the graph of the inverse function f^{-1}. For convenience (and as a hint), the graph of $y=x$ is also given.

32.



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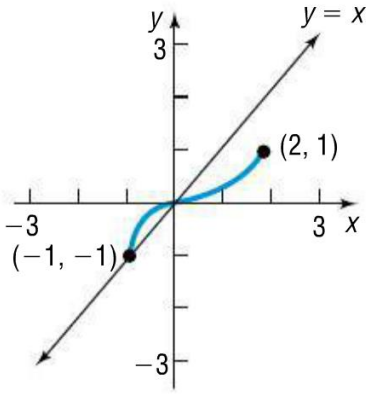

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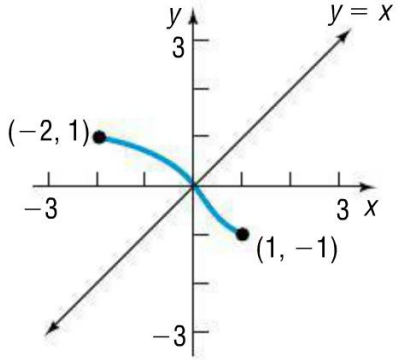

35. Taking into account reaction time, the distance d (in feet) that a car requires to come to a complete stop while traveling r miles per hour is given by the function $d(r) = 6.97r - 90.39$.

a) Express the speed r at which the car is traveling as a function of the distance d required to come to a complete stop. (All you are doing is solving the equation for r. In a story problem, that's all finding the inverse is – solving for the other variable!)

b) Verify that $r = r(d)$ is the inverse of $d = d(r)$ by showing that $r(d(r)) = r$ and $d(r(d)) = d$. (Plug the two equations into each other like you did in problems 21-25.)

c) Predict the speed that a car was traveling if the distance required to stop was 300 feet.