Long and Synthetic Division, Remainder and Factor Theorems

Division Algorithm for Polynomials:

$$\frac{f(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)} \text{ or } f(x) = q(x)g(x) + r(x).$$

$$\text{divisor}\overline{\big)\,\text{dividend}}^{\;\text{quotient}}$$

$f(x)$ is the ***dividend,*** $g(x)$ is the ***divisor,*** $q(x)$ is the ***quotient,*** and $r(x)$ is the ***remainder.***

Dividing a Polynomial by a Polynomial

1. Arrange polynomials with exponents in descending order.
2. If there are missing terms in the dividend, either write them with 0 coefficients or leave space for them.
3. Perform the long division process until the degree of the remainder is less than the degree of the divisor.
   - Divide (first term of dividend ÷ first term of divisor)
   - Multiply (quotient from previous step × divisor)
   - Subtract (dividend – product from previous step) ← DISTRIBUTE NEGATIVE!
   - Bring Down (next term in the dividend)

a) $(x^2 - 8x - 16) \div (x + 4)$

$$x+4\overline{\big)\, x^2 - 8x - 16}^{\;x - 12}$$

$-(x^2 + 4x)$

$-12x - 16$

$-(-12x - 48)$

32

$$\boxed{x - 12 + \frac{32}{x+4}}$$

b) $(x^3 - x + 6) \div (x + 2)$

$$x+2\overline{\big)\, x^3 + 0x^2 - x + 6}^{\;x^2 - 2x + 3}$$

$-(x^3 + 2x^2)$

$-2x^2 - x$

$-(-2x^2 - 4x)$

$3x + 6$

$-(3x + 6)$

0

$$\boxed{x^2 - 2x + 3}$$

c) $(6x^3 - 11x^2 + 11x - 2) \div (2x - 3)$

$$2x-3\overline{\big)\, 6x^3 - 11x^2 + 11x - 2}^{\;3x^2 - x + 4}$$

$-(6x^3 - 9x^2)$

$-2x^2 + 11x$

$-(-2x^2 + 3x)$

$8x - 2$

$-(8x - 12)$

10

$$\boxed{3x^2 - x + 4 + \frac{10}{2x-3}}$$

d) $(2x^4 - x^3 - 5x^2 + x - 6) \div (x^2 + 2)$

$$x^2+2\overline{\big)\, 2x^4 - x^3 - 5x^2 + x - 6}^{\;2x^2 - x - 9}$$

$-(2x^4 + 4x^2)$

$-x^3 - 9x^2 + x$

$-(-x^3 - 2x)$

$-9x^2 + 3x - 6$

$-(-9x^2 - 18)$

$3x + 12$

$$\boxed{2x^2 - x - 9 + \frac{3x+12}{x^2+2}}$$

To find a quotient and remainder when a polynomial is divided by $x-c$, a shortened version of long division, called synthetic division makes the task simpler.

Steps:

1. Write the **zero**, $\boldsymbol{c}$ of the divisor $\boldsymbol{x-c}$ in a box.
2. Write the coefficients of the dividend (including 0's for any missing powers of x) to the right of the box.
3. Leave space for a row of numbers under the coefficients. Draw a horizontal line below this blank row. Bring the leading coefficient down and write it below the line.
4. Multiply the latest entry below the line by c, then write the answer above the line below the next coefficient (one column to the right).
5. Add the entry just written to the coefficient above it. Record the answer below the line.
6. Repeat steps 4 and 5 until no more entries remain in row 1.
7. The numbers in row 3 are the coefficients of the quotient, with the final number at the right being the remainder.

Examples: Use synthetic division to find each quotient and remainder.

a) $(x^3+2x^2-17x+16)\div(x-3)$

3	1	2	-17	16
	↓	3	15	-6
	1	5	-2	10
	x^2	x	const	remainder

$$\boxed{x^2+5x-2+\frac{10}{x-3}}$$

b) $(3x^3+5x^2-8x+10)\div(x+4)$

-4	3	5	-8	10
	↓	-12	28	-80
	3	-7	20	-70

$$\boxed{3x^2-7x+20+\frac{-70}{x+4}}$$

c) $(3x-9+2x^3)\div(x+2)$

$(2x^3+3x-9)\div(x+2)$

-2	2	0	3	-9
	↓	-4	8	-22
	2	-4	11	-31

$$\boxed{2x^2-4x+11+\frac{-31}{x+2}}$$

d) $(z^5-32)\div(z-2)$

2	1	0	0	0	0	-32
	↓	2	4	8	16	32
	1	2	4	8	16	0

$$\boxed{z^4+2z^3+4z^2+8z+16}$$