Continuity

When we plot function values from real life data, we often connect the points with an unbroken curve. A graph of the Dow Jones industrial average is a good example. By connecting the data points we are assuming we are working with a "continuous function". This means that the function values don't jump from one point to another. If a graph can be sketched in one motion without lifting the pencil, it is said to be continuous. Continuous functions are the cornerstone of calculus. Most physical motions, chemical reactions, and physiological responses can be modeled using continuous functions.

Look at figure 2.18. Is it continuous? Where is this discontinuous?

Notice the limit at these points.

$\lim_{x\to 1} \; does \;\; not \;\; exist$ $\qquad \lim_{x\to 2} = 1$ $\qquad f(2) = 2$

$At \;\; 0 < c < 4 \qquad \lim_{x\to c} f(x) = f(c)$

Definition: Continuity at a point

For interior points, $\lim_{x\to c} f(x)$ must exist and it must equal f(c).

For end points: left end $\lim_{x\to c^+} f(x)$ must equal f(c) right end $\lim_{x\to c^-} f(x)$ must equal f(c)

If a function is not continuous at a point "c" we say that *f* is **discontinuous** at "*c*" and "*c*" is a **point of discontinuity** of *f*

Look at example 2
What are the points of discontinuity? What if we restrict the domain to [0,1]?

Now look at figure 2.21 Here are **types of discontinuities**.
Removable ("hole")
Jump Discontinuity
Infinite Discontinuity
Oscillating discontinuity

If we say a function is **continuous on an interval** it **must be continuous at every point of the interval**. A continuous function is continuous at every point in its domain.

Example: $f(x) = \dfrac{1}{x}$ is continuous on its domain, but has a point of discontinuity at x=0.

Polynomial functions are continuous at every real number c. Rational functions are continuous at every point of their domains. They have points of discontinuity at the zeros of their denominators. The absolute value function y=|x| is continuous at every real number. Exponential, logarithmic, trigonometric, and radical functions are continuous at every point of their domains.

Properties of continuous functions

If the functions f and g are continuous at x=c, then the following combinations are continuous at x = c
Sums: f+g **Differences**: f-g **Products:** f∗g **Constant multiples:** k∗f for any number k
Quotients: f/g provided g(c)≠0

Composites of continuous functions are continuous. As with any composition, you just have to be careful that the point comes out of the first function can be plugged into the second function and that it is continuous there.

Theorem 7: **Composite of Continuous Functions**
If f is continuous at c and g is continuous at f(c), then the composite $g \circ f$ is continuous at c.

One important consequence of continuity is the "**Intermediate Value Theorem"**. (middle value theorem)

A continuous function never takes on two values without taking every value between those two.

Theorem 8: The Intermediate Value Theorem for Continuous Functions
A function y=f(x) that is continuous on a closed interval [a,b] takes on every value between f(a) and f(b). In other words, if y_0 is between f(a) and f(b), then y_0=f(c) for some c in[a,b]

Caution for graphing calculators—graphing in connected mode can hide discontinuities, especially "holes".

Example:

Is any real number exactly 1 less than its cube?

$$x = x^3 - 1 \qquad or \qquad x^3 - x - 1 = 0$$

Look for a zero value of the continuous function $f(x) = x^3 - x - 1$. The function changes sign between 1 and 2, so there must be a point c between 1 and 2 where f(c)=0.