Precalculus
2.2 Homework
Quadratic Applications

1. The price p (in dollars) and the quantity x sold of a certain product obey the demand equation
   $p = -\frac{1}{5}x + 80.$
   a) Find a model that expresses the revenue R as a function of x. (Remember that $R = xp$)
   b) What is the domain of R?
   c) What is the revenue if 300 units are sold?
   d) What quantity x maximizes revenue? What is the maximum revenue?
   e) What price should the company charge to maximize revenue?

2. The price p (in dollars) and the quantity x sold of a certain product obey the demand equation
   $x = -6p + 240 \qquad 0 < p \le 40$
   a) Express the revenue R as a function of x.
   b) What is the revenue if 12 units are sold?
   c) What quantity x maximizes revenue? What is the maximum revenue?
   d) What price should the company charge to maximize revenue?
   e) What price should the company charge to earn at least $1800 in revenue?

3. Jimmy has 540 yard of fencing and wishes to enclose a rectangular area for a garden.
   Scenario I: Jimmy needs to use the fencing on all four sides of the garden.
   a) Express the area A of the garden as a function of the width w of the rectangle.
   b) For what value of w is the area the largest?
   c) What is the maximum area?
   Scenario II: Jimmy plans to use his house as one side of the garden, so he only needs to use fencing on the other three sides.
   d) Express the area A of the garden as a function of the width w of the rectangle.
   e) For what value of w is the area the largest?
   f) What is the maximum area?

4. A projectile is fired from a cliff 150 feet above the water at an inclination of 45° to the horizontal, with a muzzle velocity of 40 feet per second. The height h of the projectile above the water (in feet) is modeled by $h(x) = \dfrac{-32x^2}{40^2} + x + 150,$ where x is the horizontal distance of the projectile from the face of the cliff.
   a) At what horizontal distance from the face of the cliff is the height of the projectile a maximum?
   b) Find the maximum height of the projectile.
   c) At what horizontal distance from the face of the cliff will the projectile strike the water?
   d) When the height of the projectile is 50 feet above the water, how far is it from the cliff?

5. A suspension bridge with weight uniformly distributed along its length has twin towers that extend 75 meters above the road surface and are 400 meters apart. The cables are parabolic in shape and are suspended from the tops of the towers. The cables touch the road surface at the center of the bridge. Find the height of the cables at a point 100 meters from the center of the bridge. (Assume the road is level.)

6. A rain gutter is to be made of aluminum sheets that are 12 inches wide by turning up the edges 90°. See the illustration. What depth will provide maximum cross-sectional area and hence allow the most water to flow? What is the maximum cross-sectional area?
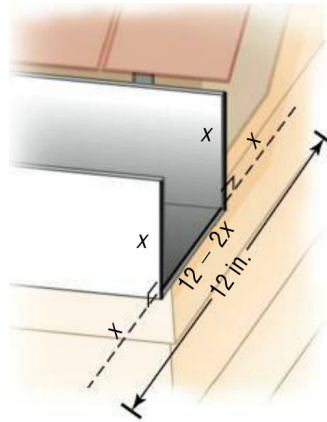
Extra Credit: A Norman window has the shape of a rectangle surmounted by a semicircle of diameter equal to the width of the rectangle. See the figure. If the perimeter of the window is 18 feet, what dimensions will admit the most light (maximize the area)? What is the maximum area?
(Hint: Circumference of a cirle $= 2\pi r$; Area of a circle $= \pi r^2$)