Vectors in the Plane

In some quantities we deal with, only the magnitude (value or number) is important. Your speedometer tells you how fast you are going. These are called scalars. If we are concerned about both the magnitude and direction, then we have a quantity called a vector. Scalar- only magnitude is important
Vector – magnitude and direction are important. There are many ways to represent a vector. We will learn several; the first is a geometric representation.

Definitions: Two-Dimensional Vector
A **two-dimensional vector v** is an ordered pair of real numbers, denoted in **component form** as <a, b>. The numbers a and b are the **components** of the vector **v**. The **standard representation** of the vector <a, b> is the arrow from the origin to the point (a,b). The **magnitude** (or **absolute value**) of **v**, denoted $|\boldsymbol{v}|$, is **the length** of the arrow, and the **direction of v** is the direction in which the arrow is pointing. The vector **0** = <0, 0>, called the **zero vector**, has zero length and no direction.

Magnitude of a Vector: The **magnitude** or **absolute value** of the vector <a,b> is the nonnegative real number $|\langle a, b\rangle| = \sqrt{a^2 + b^2}$.

Direction Angle of a Vector: The **direction angle** of a nonzero vector **v** is the smallest nonnegative angle **θ** formed with the **positive x-axis** as the initial ray and the standard representation of **v** as the terminal ray.

If an arrow has initial point (x_1,y_1) and terminal point (x_2,y_2), it represents the vector $< x_2 - x_1, y_2-y_1>$

Example 1

Example 2

Definition: Vector Addition and Scalar Multiplication
Let $\boldsymbol{u} = \langle \mathrm{u}_1, \mathrm{u}_2\rangle and\ \boldsymbol{v} = \langle \mathrm{v}_1, \mathrm{v}_2\rangle$ be vectors and let **k** be a real number (scalar).
The **sum** or **resultant** of the vectors **u** and **v** is the vector $\boldsymbol{u}+\boldsymbol{v} = \langle \mathrm{u}_1 + \mathrm{v}_1, \mathrm{u}_2 + \mathrm{v}_2\rangle$
The **product of the scalar k** and the vector **u** is $k\boldsymbol{u} = k\langle \mathrm{u}_1, \mathrm{u}_2\rangle = \langle k\mathrm{u}_1, \mathrm{ku}_2\rangle$
The **opposite of a vector v** is **–v = (-1)v**. We define vector subtraction by $\boldsymbol{u} - \boldsymbol{v} = \boldsymbol{u} + (-\boldsymbol{v})$
Whenever we add or subtract vectors the result is called the **resultant vector.** Do you remember the parallelogram method?

Unit Vectors
u with length $|\mathbf{u}|$ **= 1** is a **unit vector.** If **v** is not the zero vector <0, 0>, then the vector $\boldsymbol{u} = \frac{\boldsymbol{v}}{|\boldsymbol{v}|}$ is a **unit vector in the direction of v.** Unit vectors provide a way to represent the direction of any nonzero vector. Any vector in the direction of **v**, or the opposite direction, is a scalar multiple of this unit vector **u**.
The values correspond exactly to sin θ and cos θ.
In general $\boldsymbol{v} = \langle \boldsymbol{a}, \boldsymbol{b}\rangle; \boldsymbol{a} = |\boldsymbol{v}|\boldsymbol{cos\theta}\ \boldsymbol{and}\ \mathbf{b} = |\boldsymbol{v}|\boldsymbol{sin\theta}$

Example 3

Look at the Properties of Vector Operations on page 547.

Example 4 Finding Ground Speed and Direction

Example 5 Doing Calculus Componentwise

Velocity, Acceleration, and Speed
Suppose a particle moves along a smooth curve in the plane so that its position at any time t is (x(t), y(t)), where x and y are differentiable functions of t.
Particle's position vector is $r(t) = \langle x(t), y(t) \rangle$
Particle's velocity vector is $v(t) = \langle \frac{dx}{dt}, \frac{dy}{dt} \rangle$
Particle's speed is the magnitude of **v**, denoted by $|\boldsymbol{v}|$**.** Speed is scalar, not a vector.
Particle's acceleration vector is $a(t) = \langle \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2} \rangle$
Particle's direction of motion is the **direction vector** $\frac{\boldsymbol{v}}{|\boldsymbol{v}|}$

Example 6

Example 7

Example 8

Displacement and Distance Traveled
Suppose a particle moves along a path in the plane so that its velocity at any time t is $\boldsymbol{v}(t) = (\mathrm{v}_1(t), \mathrm{v}_2(t))$ **,** v_1 and v_2 are integrable functions of t.
The **displacement** from t = a to t = b is given by the vector $\langle \int_a^b v_1(t)dt,\ \int_a^b v_2(t)dt \rangle$
The preceding vector is added to the position at time t = a to get the position at time t = b.
The **distance traveled** from t= a to t = b is $\int_a^b |v(t)| dt = \int_a^b \sqrt{(v_1(t))^2 + (v_2(t))^2}\ dt$

Example 9

Example 10