

Probability with Counting Rules

The *sample space* S of a chance process is the set of all possible outcomes.

Event: Any outcome or collection of outcomes from some chance process is called an *event*. An event is a subset of the sample space. Events are usually designated by capital letters. We write the probability of event A as $P(A)$.

Chance Process	Sample Space	Event
Flip a coin	$S = \{\text{heads, tails}\}$	$B = \{\text{heads}\}$
Roll a die	$S = \{1, 2, 3, 4, 5, 6\}$	Even numbers $E = \{2, 4, 6\}$
Pick a letter in the word "probability"	$S = \{P, R, O, B, A, I, L, T, Y\}$	Vowels $V = \{O, A, I, Y\}$

Probability: A number between 0 and 1 that describes the long-run proportion of repetitions of the chance process on which an event occurs. An event with probability 0 never occurs, and an event with probability 1 occurs on every repetition. Probability is often interpreted as the *likelihood* of an event occurring.

If all outcomes in the sample space are equally likely, the *probability* that event A occurs is

$$P(A) = \frac{\# \text{ of outcomes corresponding to event } A}{\text{total } \# \text{ of outcomes in the sample space}}$$

★ **Note:** This formula only works if all the outcomes in the sample space are equally likely, which is not always true!

Many probability problems come down to counting the number of ways an event can happen and the number of outcomes in the sample space. This can either be done by making a list of the sample space or by using the Fundamental Counting Rule, permutations, and combinations.

Example: Give the sample space for flipping a coin three times. If a coin is flipped three times, what is the probability that it lands on heads at least twice?

Example: Give the sample space for rolling two dice. If two dice are rolled, what is the probability that the sum will be 8 or 9?

Example: If license plate numbers consisting of three letters followed by three numbers are assigned at random, what is the probability that the plate number will end with 911? (Assume letters and numbers can be repeated and all sequences are allowed.)

Example: A vocabulary section on a test consists of 6 words that need to be matched with their definitions. If you have no idea what any of the words mean and are completely guessing, what is the probability that you will match all 6 of the words with the correct definitions?

Examples: An urn contains 1 green, 4 red, and 5 yellow marbles. Two marbles are selected at random.
a. How many different sets of two marbles can be selected from the urn?

Find the probability of each selection. Express your answers as simplified fractions.

b. $P(2 \text{ red})$

c. $P(1 \text{ red and } 1 \text{ yellow})$

d. $P(1 \text{ green and } 1 \text{ yellow})$

e. $P(2 \text{ yellow})$

Example: A drawer contains 3 blue pens and 7 black pens. I reach in and draw 5 pens at random. What is the probability that exactly 2 of the pens are blue?

Example: A small voting district has 101 female voters and 95 male voters. A random sample of 10 voters is drawn. What is the probability exactly 7 of the voters will be female?

Examples: A standard deck of playing cards contains 52 cards: 13 each of spades (black), clubs (black), hearts (red), and diamonds (red). The 13 cards in each suit are the 2, 3, 4, 5, 6, 7, 8, 9, 10, the three face cards: the jack (J), queen (Q), and king (K), and the ace (A).

a. What is the probability that a hand of 7 cards will include 3 hearts, 2 diamonds, a club, and a spade?

b. If a hand of 5 cards is selected at random, what is the probability of a flush (all the same suit)?

c. What is the probability that a hand of 5 cards will include four of a kind (all the same number)?

Example: Powerball is a multi-state lottery. Players choose five different numbers from 1 to 69 and one Powerball number from 1 to 26. Twice per week, 5 white balls are drawn randomly from a drum with 69 white balls, numbered 1 to 69, and then one red Powerball is drawn randomly from a drum with 26 red balls, numbered 1 to 26. A player wins the jackpot by matching all five numbers drawn from the white balls in any order and matching the number on the red Powerball. What is the probability of winning the jackpot with one \$2 Powerball ticket?