The Binomial Theorem

Binomial Coefficients (also known as ***combinations***): $\binom{n}{j} = \frac{n!}{j!(n-j)!}$. The symbol $\binom{n}{j}$ is read "n choose j."

It can also be written as $C(n, j)$, C_j^n, or ${}_nC_j$. This expression is most commonly used in probability. It is the number of ways of selecting distinct groups of j objects from a group of n objects. For example, "How many different groups of 5 students can be selected from a class of 30 students?"

Examples: Evaluate the following expressions.

a. $\binom{5}{2}$ b. $\binom{9}{3}$ c. $\binom{27}{1}$ d. $\binom{50}{17}$

Pascal's Triangle:

Row 0:										1										
Row 1:									1		1									
Row 2:								1		2		1								
Row 3:							1		3		3		1							
Row 4:						1		4		6		4		1						
Row 5:					1		5		10		10		5		1					
Row 6:				1		6		15		20		15		6		1				
Row 7:			1		7		21		35		35		21		7		1			
Row 8:		1		8		28		56		70		56		28		8		1		
Row 9:	1		9		36		84		126		126		84		36		9		1	

etc.

Pascal's triangle is a handy way of organizing binomial coefficients. The sides consist of all 1's, and any other entry is the sum of the two nearest entries in the row above. The top row (Row 0) consists of $\binom{0}{0}$, the next row $\binom{1}{0}$ and $\binom{1}{1}$, the next $\binom{2}{0}$, $\binom{2}{1}$, and $\binom{2}{2}$, etc. Row n consists of $\binom{n}{j}$ from $j = 0$ to $j = n$.

Binomial Theorem: Let a and b be real numbers. For any positive integer n,

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{j}a^{n-j}b^j + \ldots + \binom{n}{n}b^n = \sum_{j=0}^{n}\binom{n}{j}a^{n-j}b^j$$

The coefficients are the values on row n of Pascal's triangle.

Examples: Use the binomial theorem to expand the following expressions:

a. $(x+4)^4$

b. $\left(3x-2\right)^5$

c. $\left(x^2+3y\right)^3$

d. $\left(\sqrt{x}-\sqrt{3}\right)^6$

Finding a Particular Coefficient or Term in a Binomial Expansion

Based on the expansion of $\left(a+b\right)^n$, the term containing a^j is $\binom{n}{n-j}a^j b^{n-j}$.

Examples:

a. Find the coefficient of x^5 in the expansion of $\left(x-1\right)^8$.

b. Find the coefficient of y^3 in the expansion of $\left(4y+2\right)^6$.

c. Find the fifth term in the expansion of $\left(2x-y\right)^9$.

d. Find the third term in the expansion of $\left(\sqrt{x}+1\right)^7$.