## Arithmetic Sequences

An arithmetic sequence is one in which the difference between successive terms of a sequence is always the same number.

An arithmetic sequence may be defined recursively as $a_{1}=a, a_{n}=a_{n-1}+d$, where $a$ is the first term and $d$ is the common difference. The terms of an arithmetic sequence with first term $a_{1}$ and common difference $d$ follow the pattern $a_{1}, a_{1}+d, a_{1}+2 d, a_{1}+3 d, \ldots, a_{1}+(n-1) d$, where $d=a_{n}-a_{n-1}$.

Examples: Determine whether the following sequences are arithmetic:
a) $3,7,11,15,19, \ldots$
b) $\frac{7}{3}, \frac{5}{3}, 1, \frac{1}{3}, \ldots$
$\star$ To show that a sequence is arithmetic, find $a_{n}$ and $a_{n-1}$. If $a_{n}-a_{n-1}$ is a constant (does not have a variable), then the sequence is arithmetic.

Examples: Show that the following sequences are arithmetic and find the common difference.
a) $\left\{s_{n}\right\}=\{2 n-4\}$
b) $\left\{b_{n}\right\}=\left\{\ln 2^{n}\right\}$
$\boldsymbol{n}$ th Term of an Arithmetic Sequence: For an arithmetic sequence $\left\{a_{n}\right\}$ whose first term is $a_{1}$ and whose common difference is $d$, the $n$th term is determined by the formula $a_{n}=a_{1}+(n-1) d$.

Examples: Find the $n$th term and the fifty-first term of the following sequences.
a) $a_{1}=6, d=-2$
b) $a_{1}=1, d=-1 / 3$

Examples: Find the indicated term in each arithmetic sequence.
a) 80 th term of $29,26,23,20, \ldots$
b) 86 th term of $2, \frac{5}{2}, 3, \frac{7}{2}, \ldots$

Examples: Find the first term and common difference of the arithmetic sequence described. Give a recursive formula for the sequence, and write a formula for the $n$th term.
a) 4 th term is 3 , 20 th term is 35
b) 5 th term is 30,13 th term is -2

## Sum of an Arithmetic Sequence

The sum $S_{n}$ of the first $n$ terms of an arithmetic sequence $\left\{a_{n}\right\}$ with first term $a_{1}$ and common difference $d$ is given by $S_{n}=a_{1}+a_{2}+a_{3}+\ldots+a_{n}=\frac{n}{2}\left(a_{1}+a_{n}\right)$.

Examples: Find each sum.
a) $-1+3+7+\ldots+(4 n-5)$
b) $1+3+5+\ldots+59$
c) $7+1-5-11-\ldots-299$
d) $\sum_{k=1}^{90}(3-2 k)$
e) $\sum_{k=1}^{80}\left(\frac{k}{3}+\frac{1}{2}\right)$

Example: The corner section of a football stadium has 15 seats in the first row and 40 rows in all. Each successive row contains two additional seats. How many seats are in this section?

