## **Arithmetic Sequences**

An *arithmetic sequence* is one in which the difference between successive terms of a sequence is always the same number.

An arithmetic sequence may be defined recursively as  $a_1 = a$ ,  $a_n = a_{n-1} + d$ , where *a* is the first term and *d* is the *common difference*. The terms of an arithmetic sequence with first term  $a_1$  and common difference *d* follow the pattern  $a_1$ ,  $a_1 + d$ ,  $a_1 + 2d$ ,  $a_1 + 3d$ ,...,  $a_1 + (n-1)d$ , where  $d = a_n - a_{n-1}$ .

**Examples:** Determine whether the following sequences are arithmetic:

a) 3, 7, 11, 15, 19,	b) $\frac{7}{3}, \frac{5}{3}, 1, \frac{1}{3}, \dots$
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★ To show that a sequence is arithmetic, find  $a_n$  and  $a_{n-1}$ . If  $a_n - a_{n-1}$  is a constant (does not have a variable), then the sequence is arithmetic.

**Examples:** Show that the following sequences are arithmetic and find the common difference. a)  $\{s_n\} = \{2n-4\}$  b)  $\{b_n\} = \{\ln 2^n\}$ 

*n*th Term of an Arithmetic Sequence: For an arithmetic sequence  $\{a_n\}$  whose first term is  $a_1$  and whose common difference is *d*, the *n*th term is determined by the formula  $a_n = a_1 + (n-1)d$ .

**Examples:** Find the *n*th term and the fifty-first term of the following sequences. a)  $a_1 = 6$ , d = -2 b)  $a_1 = 1$ , d = -1/3

**Examples:** Find the indicated term in each arithmetic sequence.

a) 80th term of 29, 26, 23, 20,... b) 86th term of 2,  $\frac{5}{2}$ , 3,  $\frac{7}{2}$ ,...

**Examples:** Find the first term and common difference of the arithmetic sequence described. Give a recursive formula for the sequence, and write a formula for the *n*th term.

a) 4th term is 3, 20th term is 35

b) 5th term is 30, 13th term is -2

## Sum of an Arithmetic Sequence

The sum  $S_n$  of the first *n* terms of an arithmetic sequence  $\{a_n\}$  with first term  $a_1$  and common difference *d* is

given by 
$$S_n = a_1 + a_2 + a_3 + \dots + a_n = \frac{n}{2}(a_1 + a_n).$$

**Examples:** Find each sum. a) -1+3+7+...+(4n-5) b) 1+3+5+...+59

c) 
$$7+1-5-11-...-299$$
 d)  $\sum_{k=1}^{90} (3-2k)$ 

e) 
$$\sum_{k=1}^{80} \left( \frac{k}{3} + \frac{1}{2} \right)$$

**Example:** The corner section of a football stadium has 15 seats in the first row and 40 rows in all. Each successive row contains two additional seats. How many seats are in this section?