Trigonometric Functions

In Calculus we will always measure angles in radians. Radians are real numbers; degrees are not. Formulas and functions will be much easier with radians than with degrees.
Radian comes from the word radius. It simply relates the arc length cut by an angle to the radius.

$S = r\theta$ $C = 2\pi r$

Radian Measure

The radian measure of a central angle equals the length of the arc that the central angle cuts from the unit circle. When an angle of measure θ is placed in standard position at the center of the circle of radius r, then the basic trig functions of θ are

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$

$$\csc\theta = \frac{r}{y} \qquad \sec\theta = \frac{r}{x} \qquad \cot\theta = \frac{x}{y}$$

Graphs of Trigonometric Functions

y = cos(x) y = sin (x) y = tan (x)

y = sec(x) y = csc(x) y = cot (x)

Periodicity: Periodic Function, Period: A function f(x) is periodic if there is a positive number p such that f(x+p) = f(x) for every value of x. The smallest such value of p is the period of f.
The period of a sine curve and cosine curve is 2π.
What is the period for cosecant, and secant?
What is the period for tangent and cotangent?

Many things are periodic…brain waves, seasons, heartbeats, voltage, moon phases, rotational machines.

Even and Odd Trigonometric Functions:
Sin, csc, tan, and cot are odd functions Cos, sec are even funtions
So….

sin (-x) = -sin (x)	tan (-x) = -tan (x)	cos (-x) = cos (x)
csc (-x) = -csc (x)	cot (-x) = -cot (x)	sec (-x) = sec (x)

Transformations of Trigonometric Graphs
The rules for shifting, stretching, shinking and reflecting the graph of a function apply to the trigonometric functions.

y = a *f* (b (x – c)) + d

a is the vertical stretch or shrink; reflection about x-axis
(the amplitude of f(x) = y is $|a|$) or $A = \frac{\max - \min}{2}$

f is the function, i.e. sin, cos, etc.

b is the horizontal stretch or shrink; reflection about y-axis
The period of y or f(x) is $\frac{2\pi}{|b|}$ for sin, cos, sec, csc. The period of y or f(x) is $\frac{\pi}{|b|}$ for tan & cot.

c is the horizontal shift or phase shift

d is the vertical shift

frequency is the reciprocal of the period of the wave

Example: $y = 3\sin(2x + \pi) - 4$

Trig functions are not one-to-one so we must restrict the domain.
Inverse Trigonometric Functions:
Definitions

Function	**Domain**	**Range**
$y = \cos^{-1} x$	[-1,1]	$[0, \pi]$
$y = \sin^{-1} x$	[-1,1]	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$y = \tan^{-1} x$	(-∞,∞)	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$y = \sec^{-1} x$	$x \le -1$ or $x \ge 1$	$0 \le y \le \pi, y \ne \frac{\pi}{2}$
$y = \csc^{-1} x$	$x \le -1$ or $x \ge 1$	$\frac{-\pi}{2} \le y \le \frac{\pi}{2}, y \ne 0$
$y = \cot^{-1} x$	$-\infty < x < \infty$	$0 < y < \pi$