1.5 notes Calculus

Functions and Logarithms

We have studied functions and relations with the mindset that functions are generally more useful in the study of calculus. We will soon see that there are many different types of functions. One type of functions that has interesting properties is a one-to-one function.

Definition: One-to-one Function
Horizontal Line test – The inverse of a relation is a function if and only if each horizontal line intersects the graph of the original relation in at most one point.

A function whose inverse is a function has a graph that **passes both the horizontal and vertical line tests.** Such a function is **one-to-one**, since every x is paired with unique y and every y is paired with a unique x.

Inverse function—if f is a one-to-one function with domain D and range R, then the **inverse function of f** (f^{-1}) is the function with domain R and range D.
Defined by $f^{-1}(b) = a$ if and only if $f(a) = b$

Consider $y = x^2$ $y=x^3$

Finding Inverses
Finding the inverse function – switch x and y then solve for y

$y=f(x) = 2x - 6$

$f(x) = x^2$ (Must restrict domain)

Writing f^{-1} as a function of x
1. Solve the equation y = f(x) for x in terms of y
2. Interchange x and y. The resulting formula will be $y= f^{-1}(x)$

Graphing inverse functions can be very easy using parametics.

$f(x) = x^2$	$x= t$	$y = t^2$
f-1(x) = ?	$x = t^2$	$y = t$

There are two special types of functions that we study because they are 1 to 1 and they are inverses of one another.
One is an exponential function $y = a^x$
The other is a logarithmic function $y = \log_a x$

$y = \log_b x$ (read as "y is the logarithm to the base b of x")

$y = \log_b x$ if and only if $b^y = x$
The domain of the logarithm function is $x>0$

To change log to expression $5 = \log_b 4$ or $4 = b^5$

To change expression to log $64 = 4^3$ or $3 = \log_4 64$

Example: $\log_2 8 =$

Basic Properties of Logarithms

Given M, N, and a are positive, real numbers;

r is a real number; and $a \neq 1$

$\log_a 1 = 0$ because $a^0 = 1$

$\log_a a = 1$ because $a^1 = a$

$a^{\log_a M} = M$ because $\log_a M = \log_a M$

$\log_a a^r = r$ because $a^r = a^r$

Basic Properties of Common Logarithms

Let x and y be real numbers with $x>0$

$\log 1 = 0$ because $10^0 = 1$

$\log 10 = 1$ because $10^1 = 10$

$10^{\log x} = x$ because $\log x = \log x$

$\log 10^y = y$ because $10^y = 10^y$

Basic Properties of Natural Logarithms

Let x and y be real numbers with x > 0

$\ln 1 = 0$ because $e^0=1$

$\ln e = 1$ because $e^1 = e$

$e^{\ln x} = x$ because $\ln x = \ln x$

$\ln e^y = y$ because $e^y=e^y$

Properties of Logarithmic Functions

Product Rule: $\log_a(MN) = \log_a M + \log_a N$

Quotient Rule: $\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$

Power Rule: $\log_a(M^r) = r\log_a M$

If M = N then $\log_a M = \log_a N$

If $\log_a M = \log_a N$ then M = N

Change of base formula: if $a \neq 1$; $b \neq 1$

and a, b, and M are positive real numbers

$$\log_a M = \frac{\log_b M}{\log_b a} \qquad \log_a M = \frac{\log M}{\log a} \qquad \log_a M = \frac{\ln M}{\ln a}$$