Notes 1.3 Calculus

Many of the ups and downs of the natural world can be described with functions.
The amount of money in a bank account, the bacteria in an infection, and the decay of carbon 14 in decomposing creatures, can be modeled with exponential functions.

Add: $y = f(x) = a^x$ to the parent functions list.

Exploration 1
$y = a^x$ for a = 2,3,5 in window [-5,5] by[-2,40] then [-5,5] by[-1,2]

$y = a^x$ is an exponential function with base a.

What does the graph look like for $y = a^x$ with *a* a positive real number greater than 1

What does the graph look like for $y = a^x$ with *a* a positive real number less than 1

Rules for exponents

Product of powers: $a^x \bullet a^y = a^{x+y}$

Quotient of powers: $\frac{a^x}{a^y} = a^{x-y}$

Power to a power: $\left(a^x\right)^y = a^{xy}$

Distribution of power over multiplication $a^x \bullet b^x = (ab)^x$

Distribution of power over division: $\frac{a^x}{b^x} = \left(\frac{a}{b}\right)^x$

Exponent of 1 $a^1 = a$

Exponent of 0 $a^0 = 1$

Negative exponent $a^{-x} = \left(\frac{1}{a}\right)^x = \frac{1}{a^x}$

Example 3 page 23 (exponential growth)

A typically occurring pattern of decay is the idea of half life. Look at Example 4. page 24

t=20; t=40; t=60; at t it is $5\left(\frac{1}{2}\right)^{\frac{t}{20}}$

Graph $5\left(\frac{1}{2}\right)^{\frac{t}{20}}$ to solve

Solve analytically/algebraically

Interpretation

Growth and decay model functions

$y = ka^x,\ k > 0$ is the model for growth if a>1 and decay if 0<a<1.

Consider how a calculator might make things easier. Look at example 5 page 25.

Many natural phenomena of growth and decay are best modeled by exponential functions with "e" as their base. $f(x) = Ae^x$

You will remember that when interest on an investment is compounded continuously the model function is $A = Pe^{rt}$

Example: $1000 @ 8% t=47 yrs