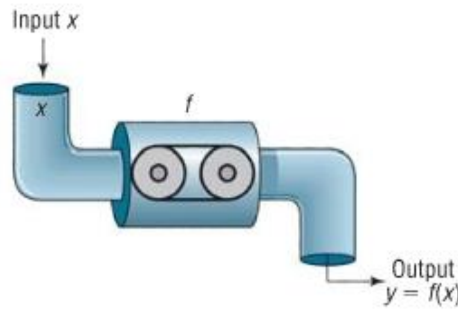
Functions Day 1.1

Relation: A correspondence between two sets, called the **domain** and the **range.**

Domain: The set of inputs (the *x*-values) of a relation.

Range: The set of outputs (the *y*-values) of a relation.

Function: A relation in which for each input there is *exactly one* output. Each element of the domain corresponds to exactly one element of the range.

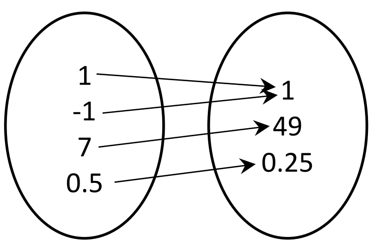


Function Machine Rules:

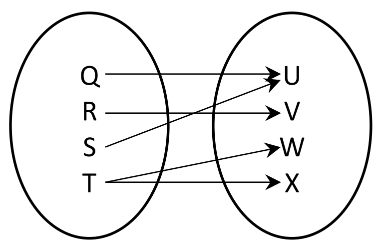
1. The machine only accepts inputs that are part of the domain.
2. The machine gets confused if there is more than one possible output for any one input. It only works if there is *only one output for each input*.

Examples: Express the relation shown in each map as a set of ordered pairs. Decide whether each relation is a function. State the domain and range.

a)



b)

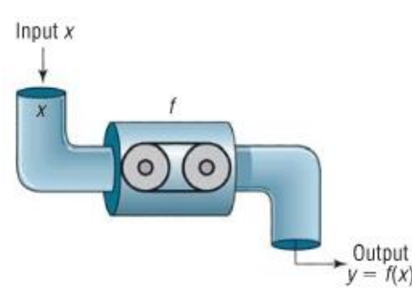


Examples: For each relation, write the domain and range and determine whether the relation is a function.

a) $\{(1,4), (3,6), (5,8), (7,10)\}$ b) $\{(2,5), (2,6), (4,7), (5,9)\}$ c) $\{(4,9), (6,7), (8,9), (10,11)\}$

Function Notation

We often use the letters f, g, and h to represent functions. The function machine to the right represents the function $y = f(x)$.



- f is the name of the function. It is the rule that relates *x* and *y*.
- x is the input, called the ***independent variable*** or ***argument.***
- y or $f(x)$ is the output, called the ***dependent variable*** (because the value of *y* *depends* on the value of *x* that is used as an input).

$f(x)$ is read "*f* of *x*" and means "the value (output) of the function *f* when the input is *x*."

$f(x)$ DOES NOT mean *f* times *x*!

Example: For the function $f(x) = -x + 9$ evaluate the following:

a) $f(2)$

b) $f(x) + f(-6)$

c) $3f(x)$

d) $f(3x)$

e) $-f(x)$

f) $f(-x)$

g) $f(x)+2$

h) $f(x+2)$

i) $f(x+h)$

Example: For the function $f(x) = 2x^2 - 4x + 3$ evaluate the following:

a) $f(2)$

b) $f(x) + f(-6)$

c) $3f(x)$

d) $f(3x)$

e) $-f(x)$

f) $f(-x)$

g) $f(x)+2$

h) $f(x+2)$

i) $f(x+h)$

Domain of a Function

The domain of a function $f(x)$ is the set of all inputs *x*.

- If the function is listed in a table or as a set of ordered pairs, the domain is the set of all first coordinates.
- If the function is described by a graph, the domain is the set of all *x*-coordinates of the points on the graph.
- If the function is described by an equation, the domain is the set of all real numbers *x* for which $f(x)$ is a real number. Figure out if there are any *x*-values that cause "problems" (zero in a denominator, square root of a negative, etc.) when you plug them into the function. If no *x*'s cause problems, the domain is all real numbers. If there are problems, the domain is all real numbers except the problem *x*'s.
- If the function is used in an application, the domain is the set of all numbers that make sense in the problem.

Tips for finding domain:

1. If the equation has fractions: denominator $\neq 0$.
2. If the equation has an even root: radicand ≥ 0. (The radicand is the stuff under the root.)
3. If you have an even root in the denominator of a fraction, you have to combine the rules above!

Examples: Determine the domain of $f(x)$.

a) $f(x)=x^2-x$

b) $f(x)=\dfrac{4x}{x^2-9}$

c) $f(x)=|x-5|$

d) $f(x)=\sqrt{x+2}$

e) $f(x)=\dfrac{1}{\sqrt{x+2}}$

f) $f(x)=\sqrt{-5x-7}$

Sums, Differences, Products, and Quotients of Two Functions

The **sum** $f+g$ is defined by $(f+g)(x)=f(x)+g(x)$

The **difference** $f-g$ is defined by $(f-g)(x)=f(x)-g(x)$

The **product** $f\cdot g$ is defined by $(f\cdot g)(x)=f(x)\cdot g(x)$

The **quotient** $\dfrac{f}{g}$ is defined by $\left(\dfrac{f}{g}\right)(x)=\dfrac{f(x)}{g(x)},\ g(x)\neq 0$

The domains of $f+g,\ f-g,$ and $f\cdot g$ consist of all numbers *x* that are in the domains of both f and g. The domain of f/g consists of all the numbers *x* for which $g(x)\neq 0$ that are in the domains of both f and g.

Example: Find the following given the functions $f(x)=\sqrt{x+5}$ and $g(x)=1-\frac{1}{x}$. Find the domain for a-d.

a) $(f+g)(x)$ b) $(f-g)(x)$ c) $(f\cdot g)(x)$ d) $\left(\frac{f}{g}\right)(x)$

e) $(f+g)(4)$ f) $(f-g)(-1)$ g) $(f\cdot g)(-5)$ h) $\left(\frac{f}{g}\right)(-4)$

Modeling and Equation solving

Mathematical Model – a mathematical structure that approximates phenomena for the purpose of studying or predicting their behavior

Numerical Models – numbers (or data) are analyzed to gain insights into phenomena

Algebraic Models – uses formulas to relate variable quantities associated with the phenomena being studied

Graphical models – visible representation of a numerical model or an algebraic model that gives insight into the relationships between variable quantities.

Fitting a curve to data. (Line of best fit)

Zero Factor Property – A product of real numbers is zero if and only if at least one of the factors in the product is zero.

Fundamental Connections:

If "a" is a real number that solves the equation f(x) = 0, then these three statements are equivalent.

1. The number "a" is a **root (or solution)** of the equation f(x) = 0
2. The number "a" is **a zero** of y = f(x)
3. The number "a" is an **x-intercept** of the graph of y=f(x).

Grapher Failure (hidden behavior)